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Mathematics and Information, Exercise sheet 7

Problem 1: (*8 points*)

Each year, with a probability of 60%, the value of a certain stock will increase by five percent, and with a probability of 40%, its value will decrease by two percent.

a) What is the expected doubling rate of this stock?

b) Which maximal/minimal growth over a period of ten years is possible?

c) Estimate the likelihood that the worth of this investment after ten years is smaller than the initial amount!

d) In order to reduce risk, it is possible to invest only some proportion $c \in [0, 1]$ of the capital and to hold back the rest. What are the answers to *a)*, *b)* and *c)* as functions of c ?

Problem 2: (*12 points*)

For your investment, you fancy two stocks. You estimate that, with a probability of 75%, the first one will increase in value by two percent, and the second one by ten percent. With a probability of five percent, the first stock will increase by two percent, whereas the second one will loose twenty percent, and with a probability of 15% both will increase by five percent. In the remaining cases, the first stock will gain ten percent, while the second one will loose twenty percent.

a) What is the probability distribution for the vector (X, Y) describing the development of those two stocks?

b) Compute the expectation and the covariance matrix of this distribution!

c) What are the marginal distributions for X and for Y, and what are their expectations and variances?

d) What's the expected doubling rate for each of the two stocks?

e) Compute the log-optimal portfolio and its doubling rate!

f) What is the maximal and the minimal value of the log-optimal portfolio after ten periods? What is the probability for each of the two cases?

g) Which expected doubling rate will you get, if for each period only one, two or three quarter of the available capital are invested?

h) How do the answers to *e)*, *f)* and *g)* change, if you regard X and Y as independent random variables with distributions given in *c)*?