

October 21, 2023

Mathematics and Information, Exercise sheet 6

Problem 1: (12 points)

You want to invest your money using two different strategies: Some part will stay on your bank account, where it doesn't earn any interest; the rest is invested in some stock which during each period with probability $1/2$ each is multiplied or divided by some constant $a > 1$.

- Determine the vector (X_1, X_2) of the random variables describing this "market"!
- By which factor is your capital multiplied during each period if you choose some portfolio $b = (b_1, b_2)$?
- Determine the growth rate of this investment!
- Determine all log optimal portfolios and their growth rate W^* !
- Describe the asymptotic behaviour of the growth rate $S_n = \prod_{i=1}^n \langle b, X \rangle$ over n periods for an arbitrary portfolio b !
- Some fairy offers you a random variable Y such that $Y = 1$ if both $X_1 \geq 1$ and $X_2 \geq 1$, and $Y = 0$ otherwise. Which optimal growth rate W^{**} can you realize by using this information?
- Check that $W^{**} - W^* \leq I(X; Y)$!

Problem 2: (4 points)

Now you find a bank paying interest for your account; during each period your capital is multiplied by some factor $c > 1$.

- Which are the log optimal portfolios for this situation?
- Under what conditions does the log optimal portfolio put all the money on one of the two alternatives?

Problem 3: (4 points)

A stock exchange described by the random variable $X = (X_1, \dots, X_m)$ with values in $\mathbb{R}_{\geq 0}^m$ and probability distribution F , adds a fund is introduced as a new product. This fund only contains stock traded locally; $c_j \in [0, 1]$ is the part of stock j for $j = 1, \dots, m$.

- Which random variables and which probability distribution describe the so extended stock exchange?
- Show that the optimal growth rate for the new stock exchange is the same as that for the old one!