

October 12, 2018

## Mathematics and Information, Exercise sheet 6

### Problem 1: (8 points)

Based on a letter count of OSCAR WILDE's *Importance of being Earnest*, the frequencies of letters in english plain text are as follows:

E	A	T	O	N	I	R	S	L	H	D	U	Y
.1182	.0833	.0803	.0771	.0740	.0708	.0609	.0578	.0508	.0452	.0340	.0332	.0327
C	M	G	W	F	B	P	K	V	J	X	Q	Z
.0314	.0285	.0221	.0203	.0180	.0169	.0150	.0120	.0102	.00395	.00185	.00131	.000123

Construct a binary HUFFMAN code for this alphabet!

### Problem 2: (5 points)

Let  $A$  be an alphabet consisting of  $n$  letters occurring with equal probability, Compute the average length of a binary HUFFMAN code for

- a)  $n = 3$     b)  $n = 24$     c)  $n = 2^k - 1$  for  $k \geq 2$

### Problem 3: (2 points)

Transmitting a message usually involves three coding steps:

- *Source coding* in order to adapt the message to the medium and possibly also compressing it
- *Channel coding* uses error correcting codes to safeguard against transmission errors
- *Cryptographic codes* safe against intelligent adversaries.

In which order should those three steps be applied for best results?

### Problem 4: (5 points)

Let  $X, Y$  be random variables with values in a subset of  $\mathbb{R}$ . Both  $X$  and  $Y$  have expectation  $\mu$  and variance  $\sigma^2$ ; their correlation is  $\rho$ . Find unit vectors  $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$  and  $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$  such that the random variables  $U = a_1X + a_2Y$  and  $V = b_1X + b_2Y$  are uncorrelated, and determine expectation and variance of  $U$  and  $V$ !