WOLFGANG K. SEILER
*Tel.* 2515

B6, B4.18

*September 20, 2018*

# Mathematics and Information, Exercise sheet 3

**Problem 1:** (*5 points*)

Let $X, Y$ be random variables with values in $A = \{0, 1\}$ and joint probability distribution $p(0,0) = \frac{1}{2}$, $p(0,1) = p(1,1) = \frac{1}{4}$ and $p(1,0) = 0$.

*a)* Find the probability distributions $p_X, p_Y$ of $X$ and $Y$!

*b)* Determine $H(X)$, $H(Y)$, $H(X,Y)$, $H(X|Y)$, $H(Y|X)$ and $I(X;Y)$!

*c)* Compute the KULLBACK-LEIBLER distances $d(p_X \| p_Y)$ and $d(p_Y \| p_X)$!

**Problem 2:** (*4 points*)

*a)* A fair dice is thrown; the random variable $X, Y, Z$ with values in $\{1,2,3,4,5,6\}$ give the numbers on top, at the bottom and on the front side. Compute the mutual informations $I(X;Y)$, $I(X;Z)$ and $I(Z;Y)$!
*Hint: The numbers on opposite sides of a dice always add up to seven.*

*b)* Determine the conditional mutual informations $I(X;Y|Z)$ and $I(X;Z|Y)$!

**Problem 3:** (*5 points*)

*a)* Let $X$ be a random variable with values in a finite Alphabet $A$, and let $f\colon A \to B$ be any mapping. Show that the entropy of the random variable $Y = f(X)$ cannot be greater than $H(X)$!
*Hint: Compute* $H\big(X, f(X)\big)$ *in two ways using the chain rule!*

*b)* Find an example where $H\big(f(X)\big) < H(X)$!

*c)* Now let $Y$ be any random variable with values in an alphabet $B$ such that $H(Y|X) = 0$, and assume that both $X$ and $Y$ take each letter from their alphabet with a positive probability. Shot that there is a mapping $f\colon A \to B$ such that $Y = f(X)$!

**Problem 4:** (*6 points*)

Let $\mathcal{X} = X_1, X_2, \ldots$ be a stationary time invariant MARKOV chain with transition matrix $A = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}$.

*a)* Determine the probability distribution and the entropy of each of the random variables $X_i$!

*b)* Compute the entropy rate of $\mathcal{X}$!

*c)* For which values of $p$ and $q$ ist this entropy rate minimal respectively maximal?

Can be submittel till Thursday, September 27, 2018 at 11.55h