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A5, C201

September 6, 2012

Mathematics and Information, Exercise sheet 1

Problem 1: (*8 points*)

a) A message source produces signs form an alphabet A consisting of six letters with respective probabilities $1/2, 1/4$, and four times $1/16$. Compute its entropy! (All logarithms should be taken to base two.)

b) Construct a binary encoding of these letters such that any sequence of letters from A has a unique encoding and the expectation value of the code length is as small as possible!

c) What changes in *a)* and *b)*, if all elements of A have equal probability?

Problem 2: (*6 points*)

A letter produced by a message source is more surprising if it has lower probability. If we want to describe this by a function $\mathrm{Sur}(p)$, we should therefor demand that $\mathrm{Sur}(p)$ is continuous and strictly decreasing. We also demand that surprises add up, i.e.

$$\mathrm{Sur}(pq) = \mathrm{Sur}(p) \cdot \mathrm{Sur}(q)\,.$$

a) Show that for every such function $\mathrm{Sur}(p)$ there exists an $a > 0$ such that $\mathrm{Sur}(p) = \log_a p$!

b) SHANNON's function $H(p_1, \dots, p_n)$ is the expectation of $\mathrm{Sur}(p)$.

Problem 3: (*6 points*)

A message source with alphabet $A = \{0, 1, 2, \dots, 10\}$ produces letters according to the following rule: It throws a fair coin at most ten times, but stops, if the coin shows *head.* If this happens in the i^{th} experiment, $i \le 10$, the letter i is sent; if it never happens, 0 is transmitted. Compute its entropy!

Hint: $\displaystyle\sum_{i=0}^{N} i q^i = \frac{nq^{n+2} - (n+1)q^{n+1} + q}{(1-q)^2}$

Abgabe bis zum Donnerstag, dem 13. September 2012, um 15.30 Uhr