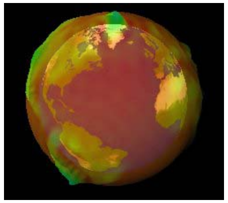
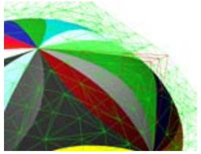


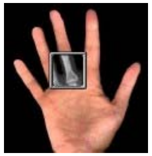
Computational Visualization

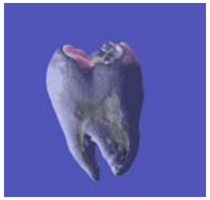
1. Sources, characteristics, representation

2. Mesh Processing

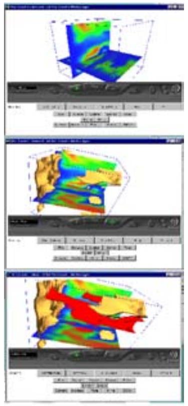


3. Contouring





4. Volume Rendering



5. Flow, Vector, Tensor Field Visualization
6. Application Case Studies

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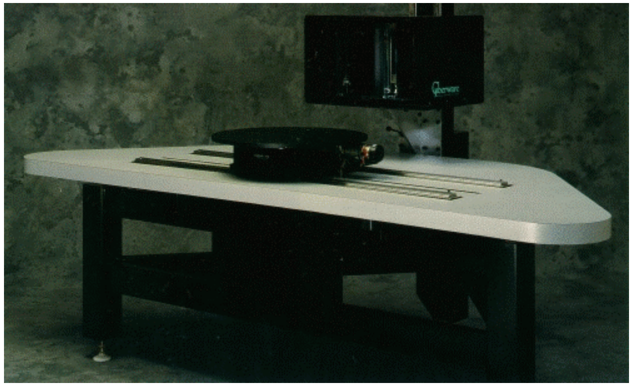
Computational Visualization: sources, characteristics and representation
Lecture 1



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Outline

- Data Sources: Meshless and Meshed
- Mesh and Field Data Characteristics
- Mesh Representations
- Mesh Finite Elements

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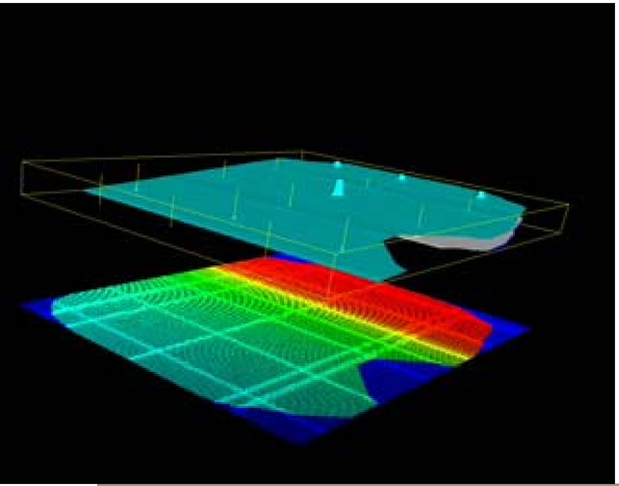
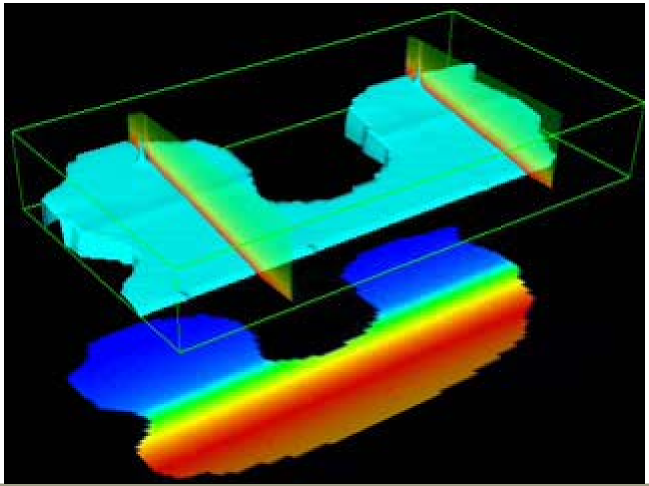
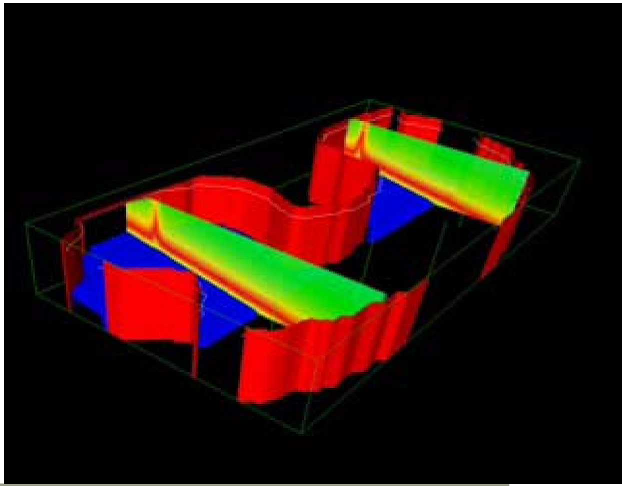
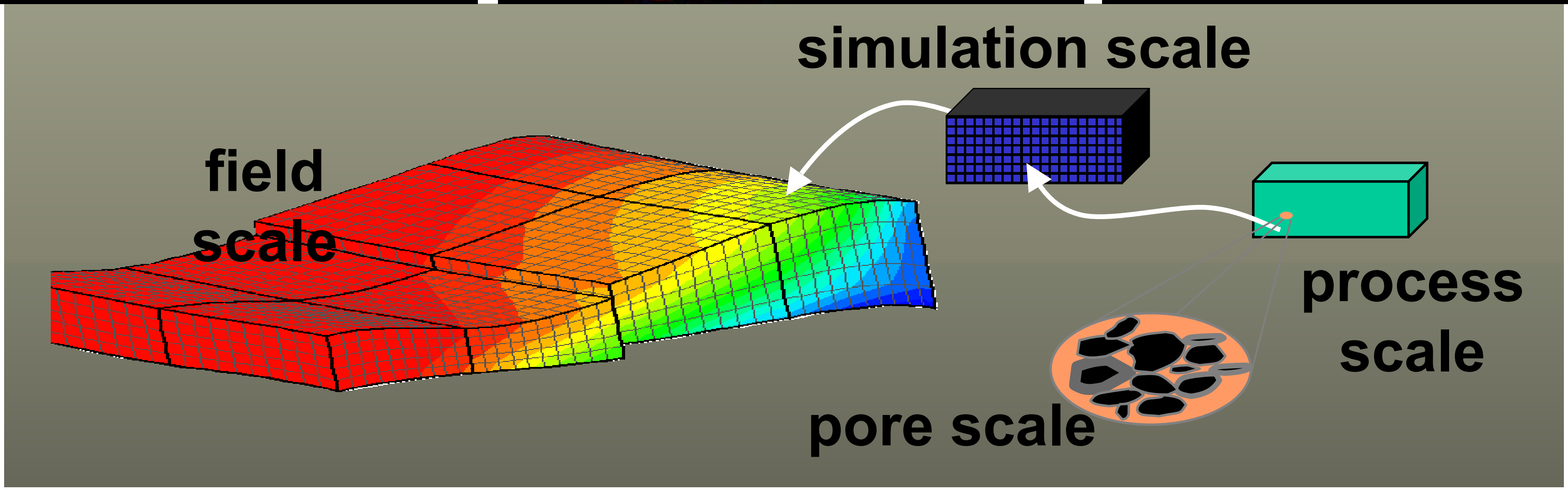
Oil Reservoir Modeling









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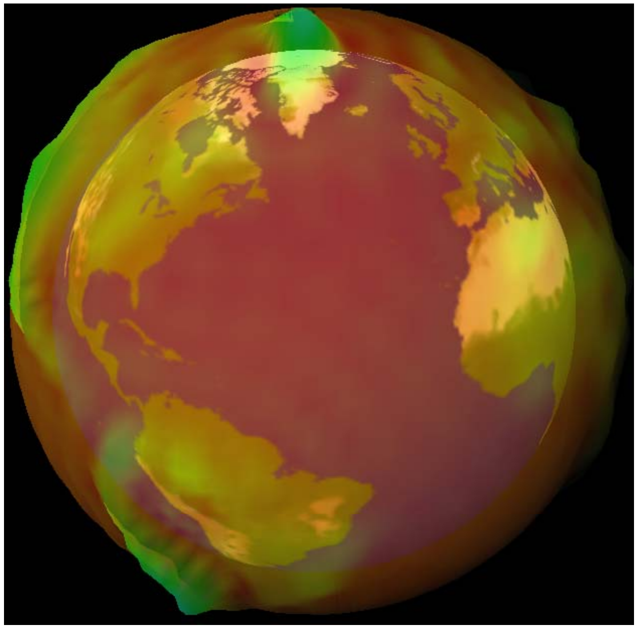
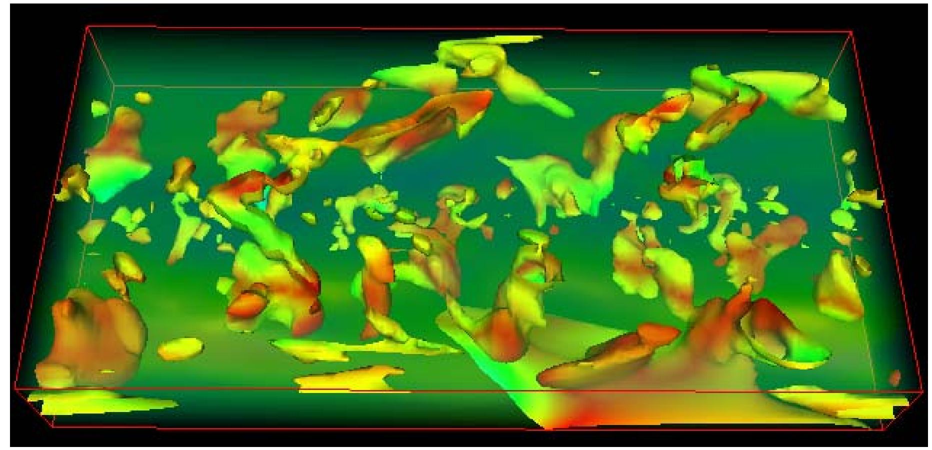
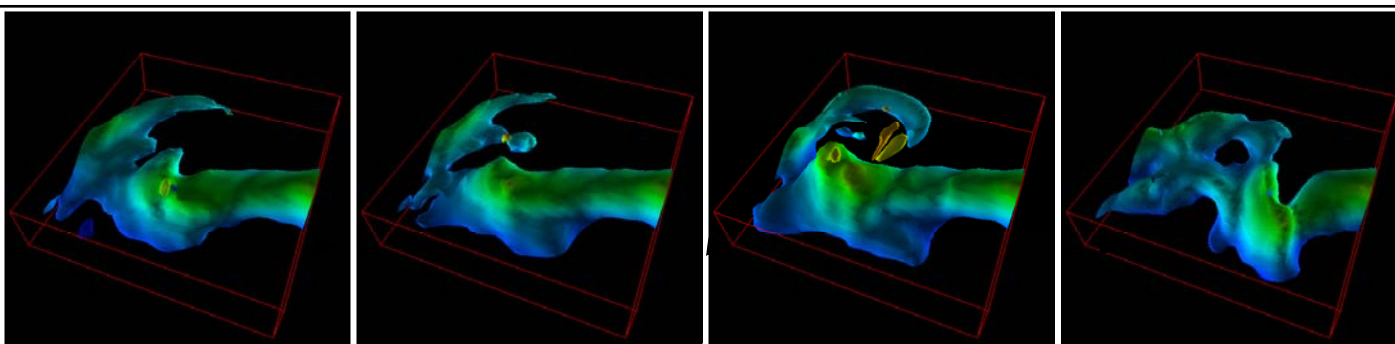
Global Climate Modeling



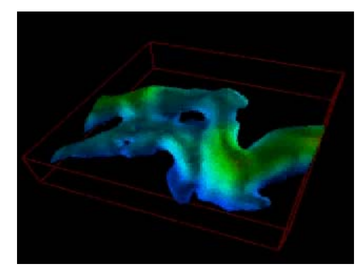


Regional Simulation

Combined Analysis



Time Evolution



Community Climate Model

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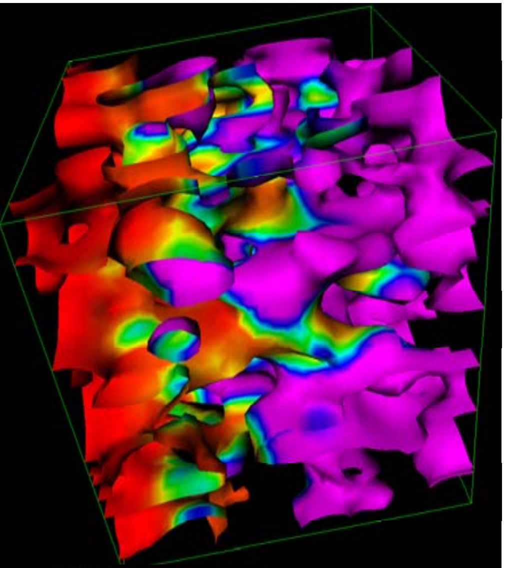
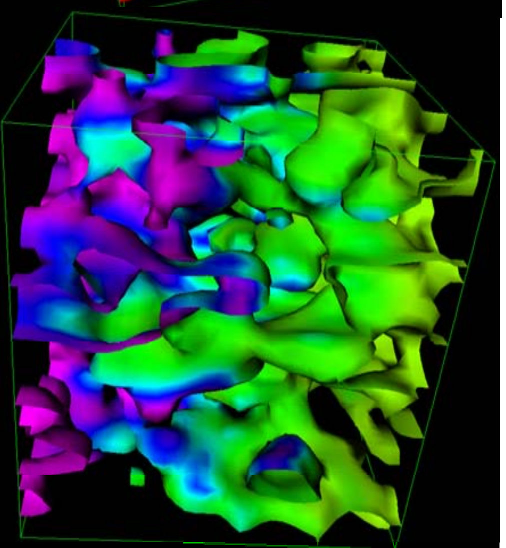
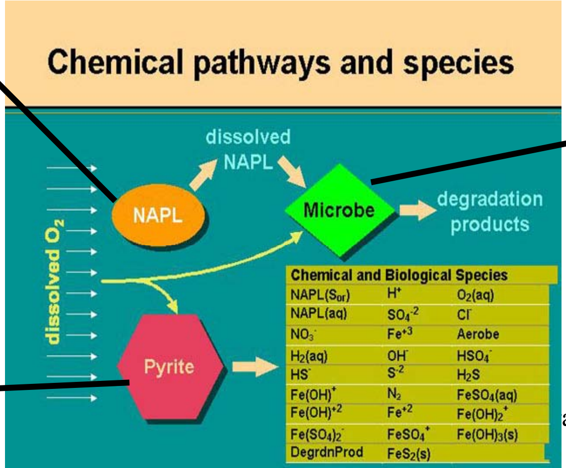
Bio-Remediation

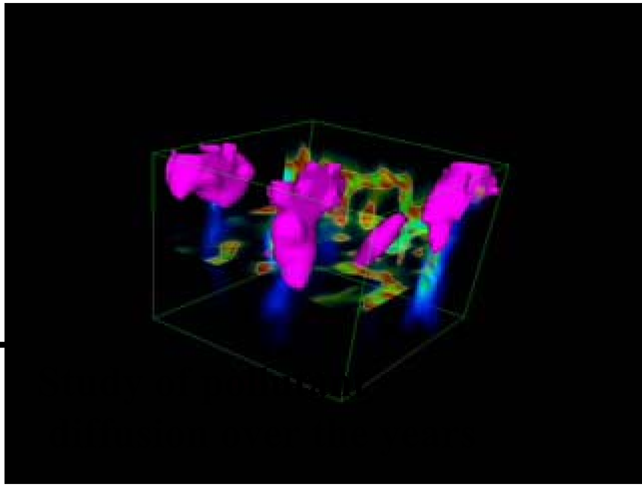
Multi-Scale Physical Simulation

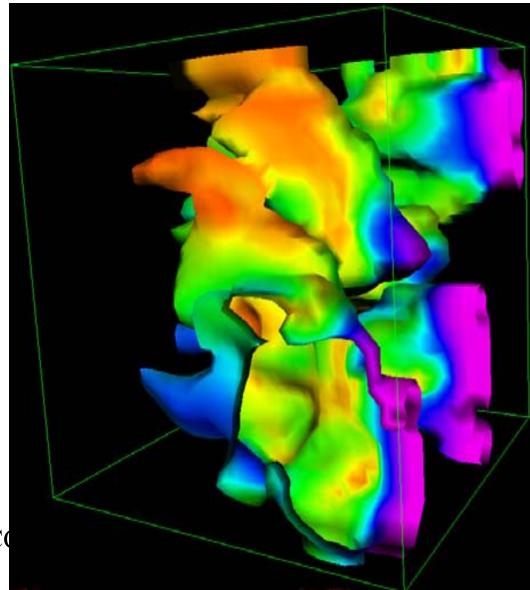




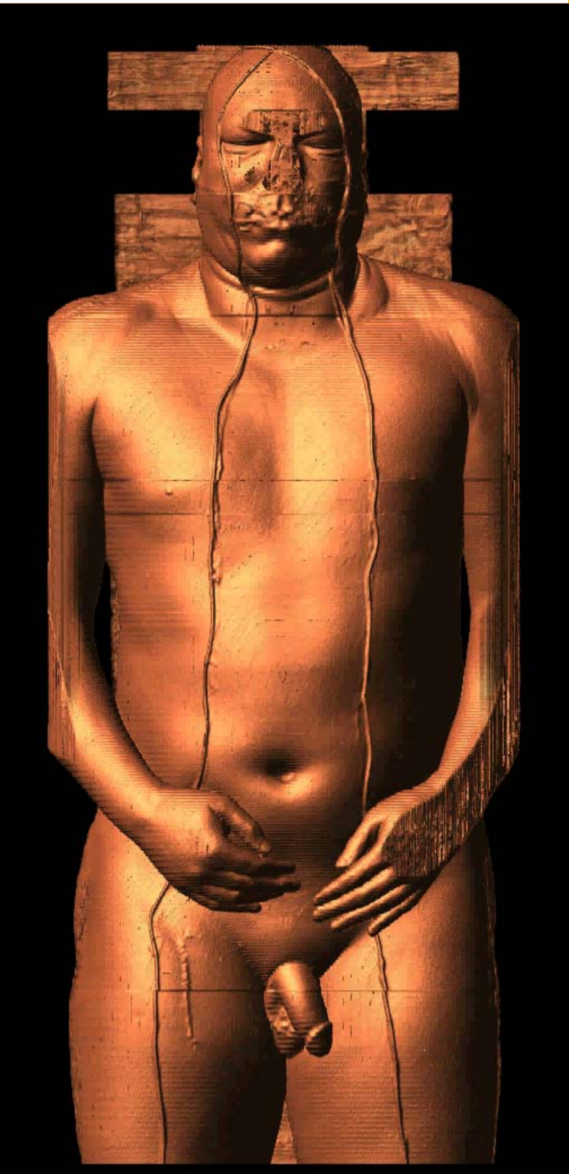


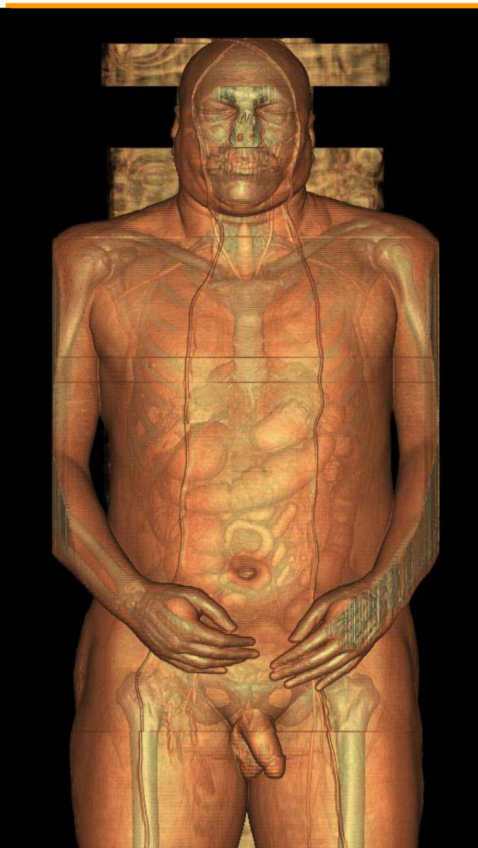
Chemical and Biological Species		
NAPL(S_{or})	H^+	O_2(aq)
NAPL(aq)	SO_4^{-2}	Cl^-
NO_3^-	Fe^{+3}	Aerobe
H_2(aq)	OH^-	HSO_4^-
HS^-	S^{-2}	H_2S
$Fe(OH)^+$	N_2	$FeSO_4$(aq)
$Fe(OH)^{+2}$	Fe^{+2}	$Fe(OH)_2^+$
$Fe(SO_4)_2^-$	$FeSO_4^+$	$Fe(OH)_3$(s)
DegrdnProd	FeS_2(s)	

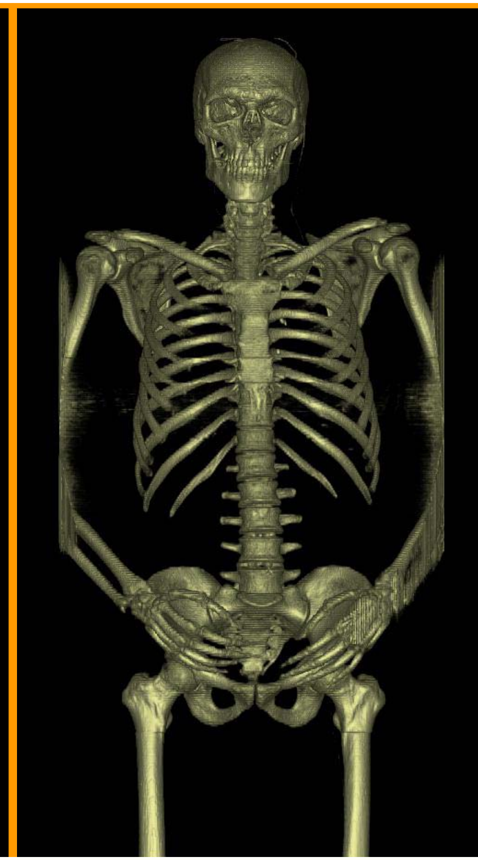




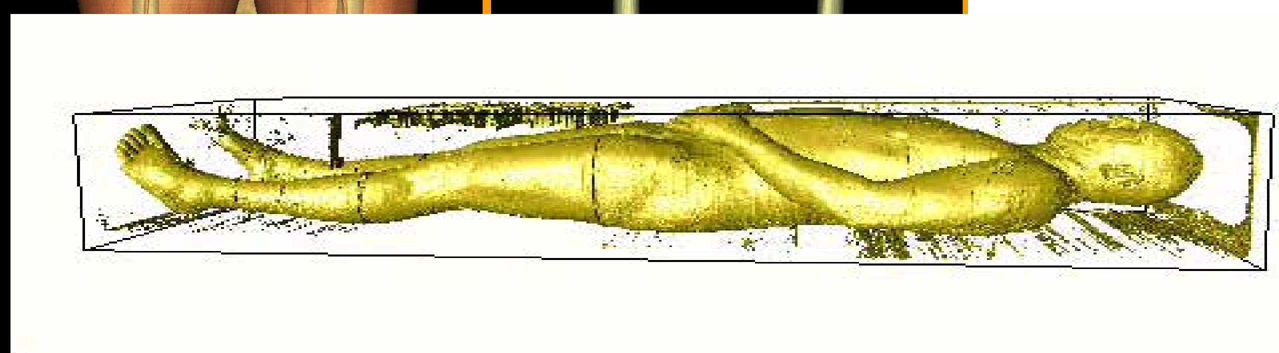
The Visible Human Project





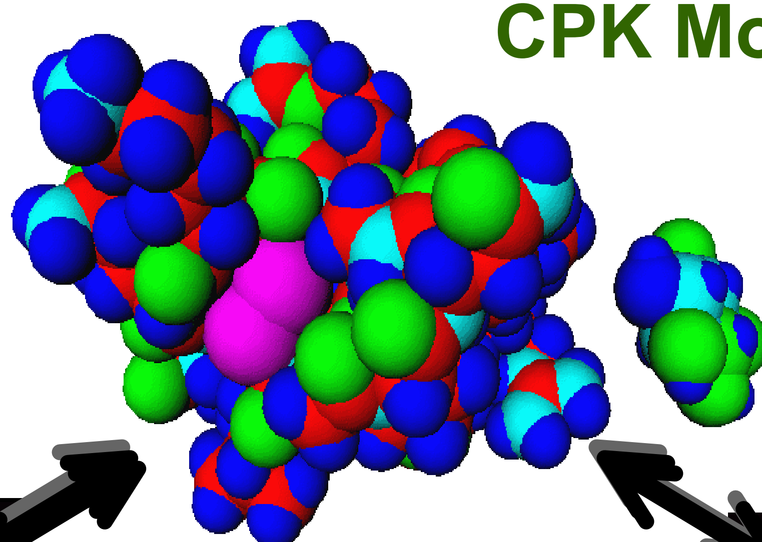


- NLM/NIMH
- Multiple modality CT, MRI, cryogenic-slices (RGB)
- Male and Female Human Cadavers imaged for research and scientific community
- Available test case for out-of-core visualization processing



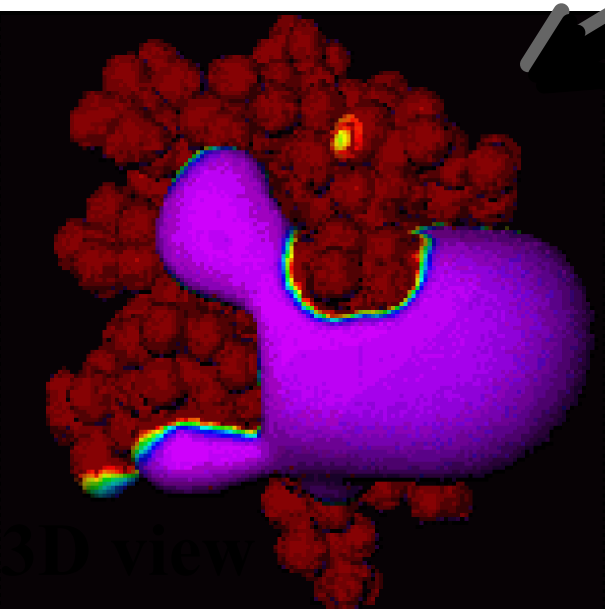
Molecular Modeling and Interactions

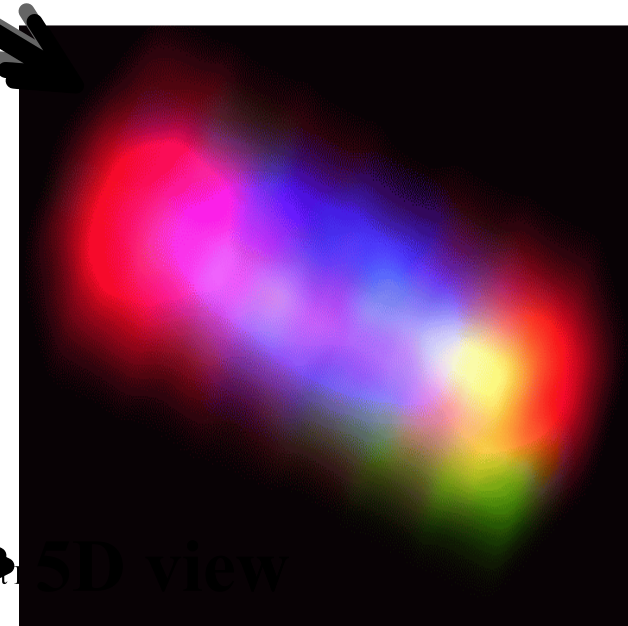
CPK Model



Molecular

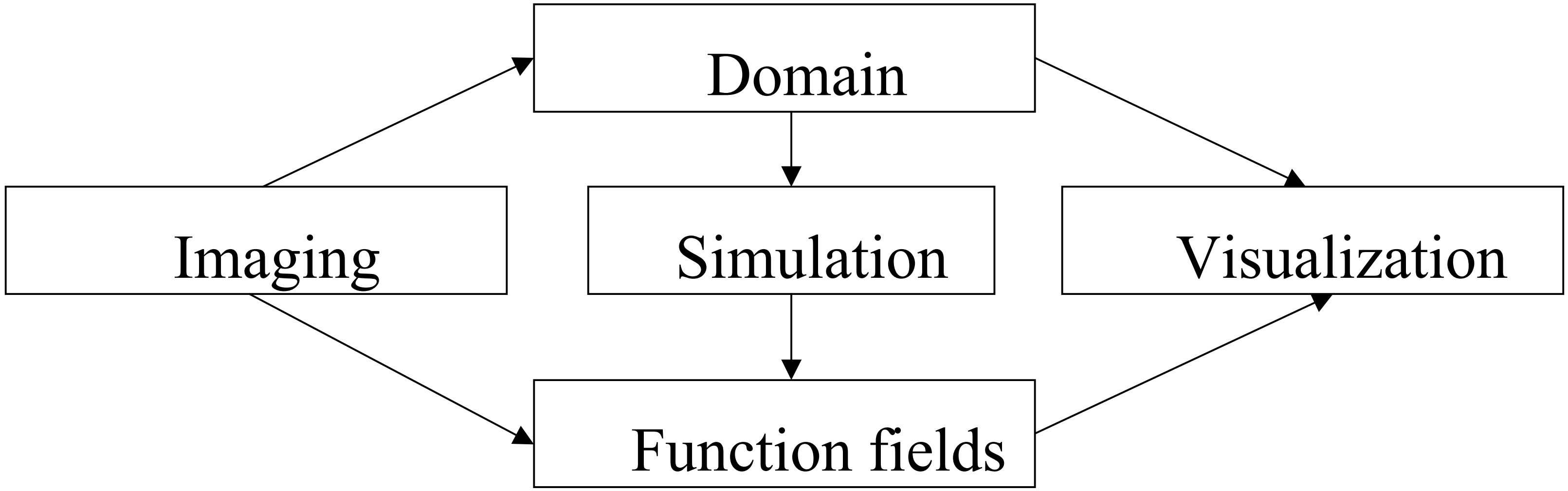
Interaction





Copyright: Cha[illegible]

Computational Visualization



•To identify and display *information* for model calibration or scientific discovery

•Support *interrogation* with quantitative queries (metric, combinatorial, topological)

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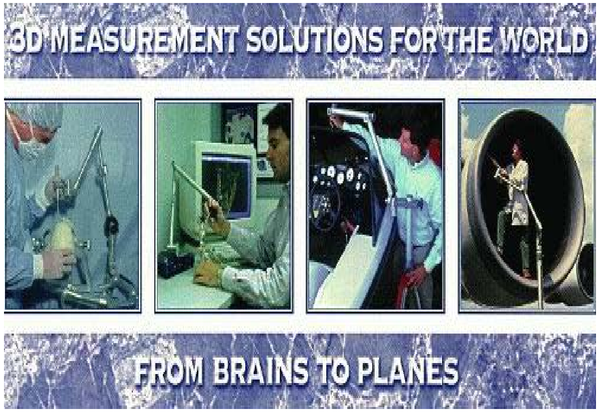
Imaging Scanners

- Scanners can yield both domains and functions on domains
  - Scanners yielding domains
    - Point Cloud Scanners: 300μ-800μ
    - CT, MRI: 10μ-200μ
    - Light microscopy: 5μ-10μ
    - Electron microscopy: < 1μ
    - Ultra microscopy like Cyro EM 50Å-100Å
  - Scanners yielding functions
    - Doppler velocimetry



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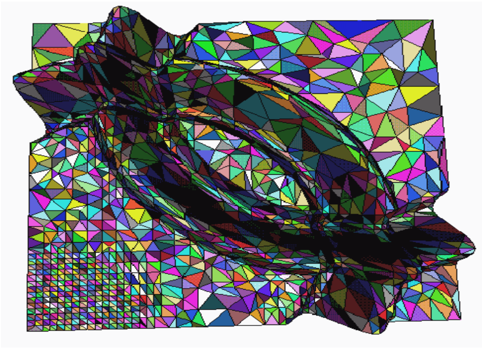
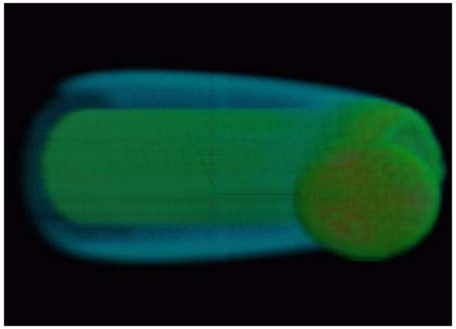
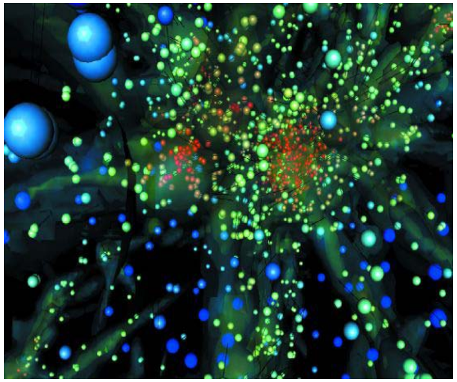
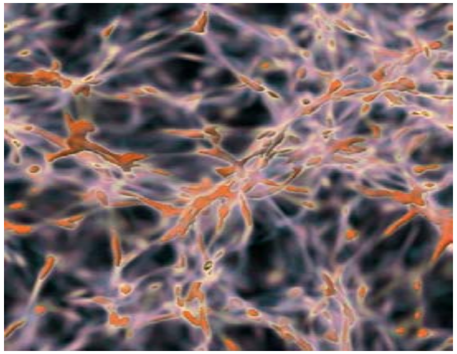
Data characteristics

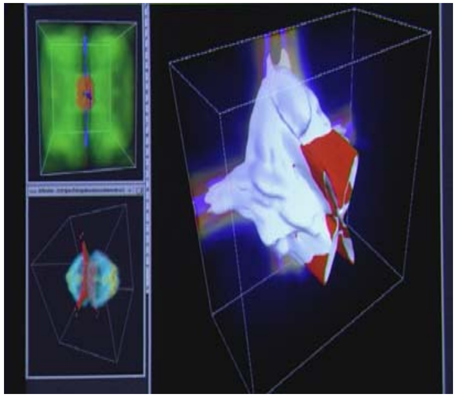
- Static
- Scalar
- Meshed
- Dense



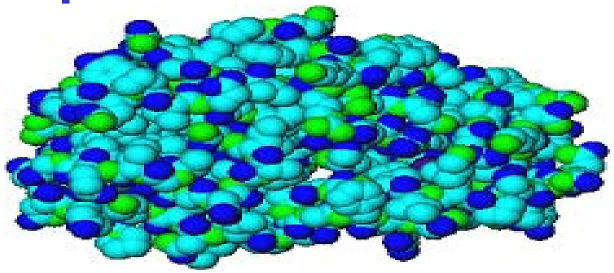


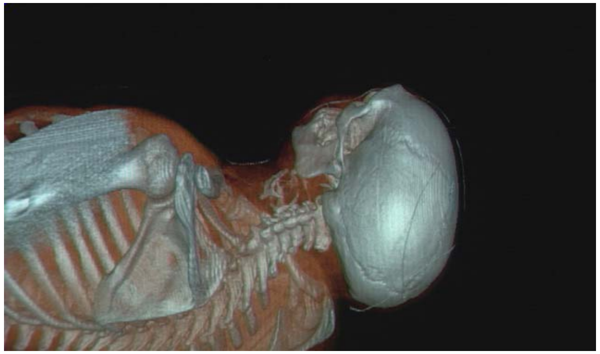






- Time varying data
- Vector , Tensor
- Meshless
- Sparse





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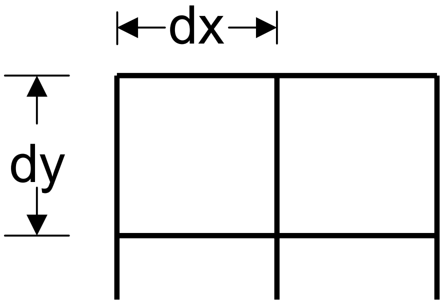
Mesh Types

- Mesh taxonomy
  - Regular static meshes:
    - There is an indexing scheme, say i,j,k, with the actual positions being determined as i*dx, j*dy, k*dz.
    - If dx=dy=dz, then,
      - In 2-D, we get a pixel, and in 3-D, a voxel.



A 2-D regular rectilinear cartesian grid

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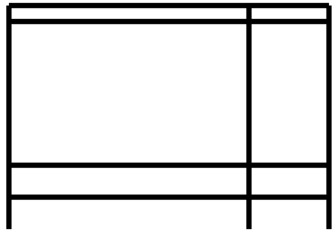
Mesh Types (contd)

– Irregular static meshes:
  - Rectilinear:
    – Individual cells are not identical but are rectangular, and connectivity is related to a rectangular grid



dx, dy are not constant in grid,
but connectivity is similar in topology
to regular grids.

A 2-D regular rectilinear grid

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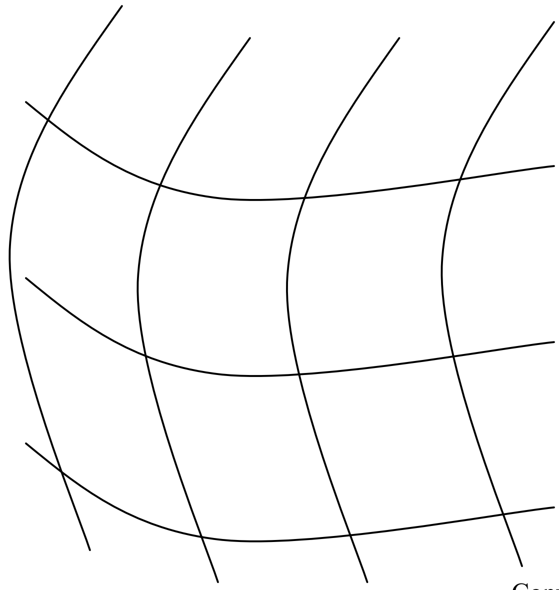
Mesh types (contd)

- Curvilinear:
  - Sometimes called structured grids as the cells are irregular cubes – a regular grid subjected to a non-linear transformation so as to fill a volume or surround an object.



A 2-D curvilinear grid

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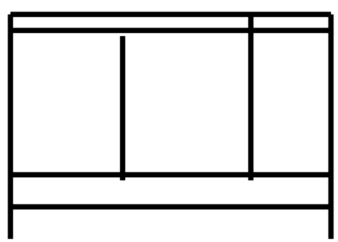
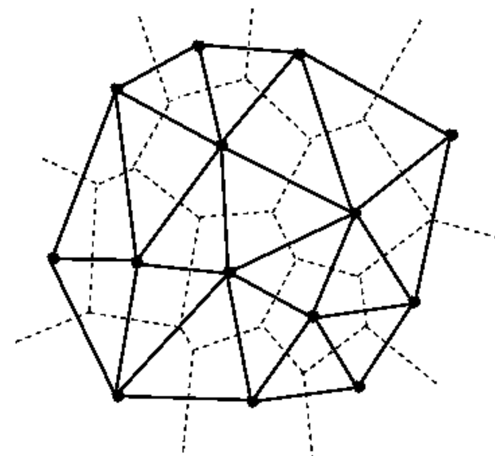
Mesh Types (contd)

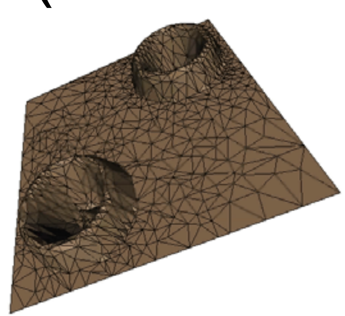
- Unstructured:
  - Cells are of any shape (tetrahedral) hexahedra, etc with no implicit connectivity – e.g. Finite element analysis





- Hybrid:
  - Combination of curvilinear and unstructured grids.

– Dynamic (Time-varying) meshes



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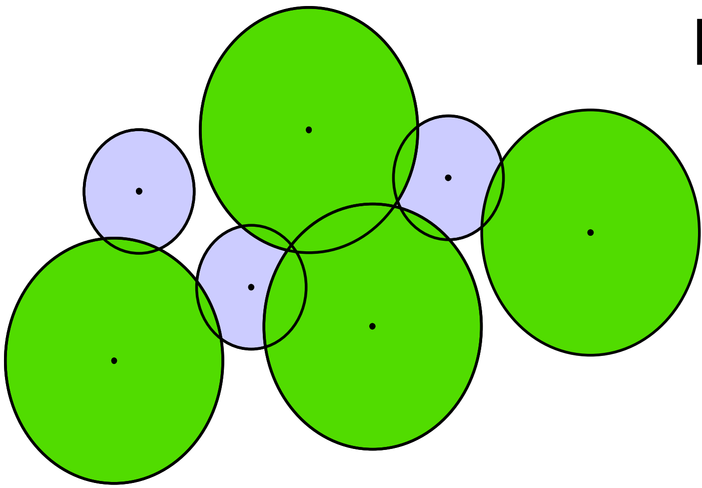
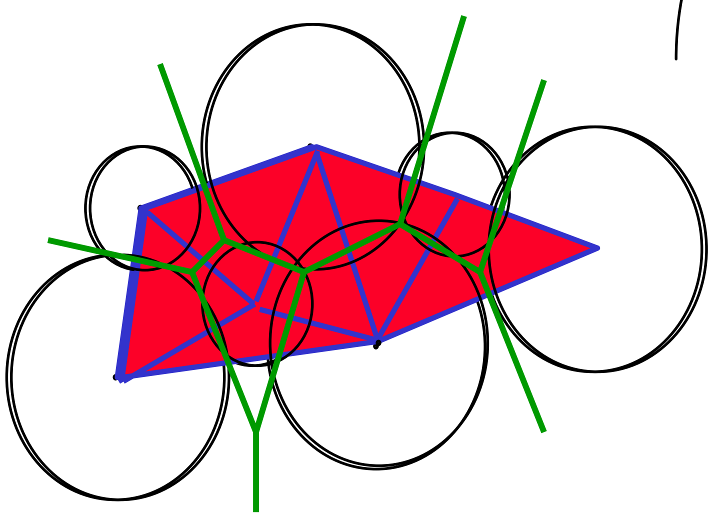
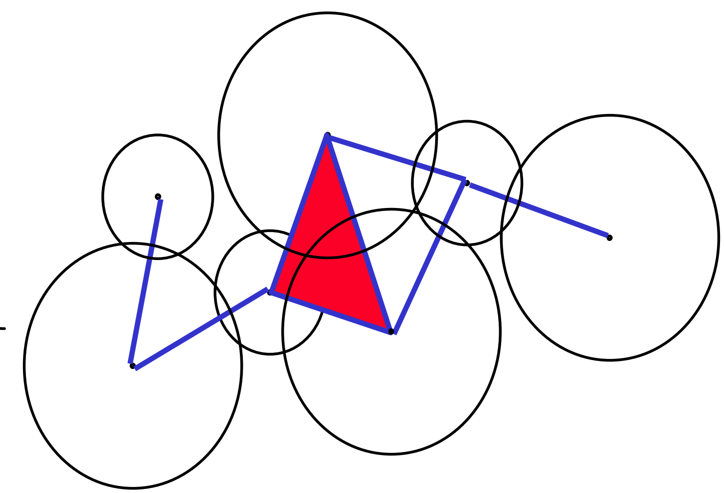
Meshless Data → Meshed Data

Triangulations (Delaunay) &
Dual Diagrams (Voronoi)



- ↗Union of balls
- ↗Triangulation & Dual
- ↗Nerve sub-complex





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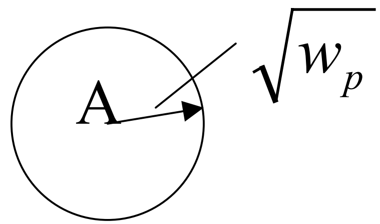
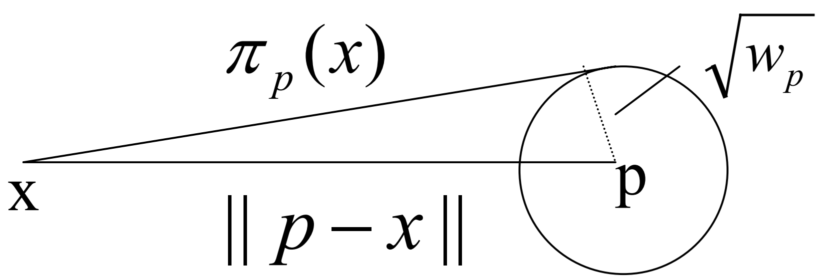
Particle Data to Meshes



Weighted point P = (p, w_p) where $p \in \Re^d, w_p \in \Re$



Power distance from $x \in \Re^d$ to p $\pi_p(x) = \| p - x \|^2 - w_p$

with $\| p - x \|^2$ is the Euclidean distance

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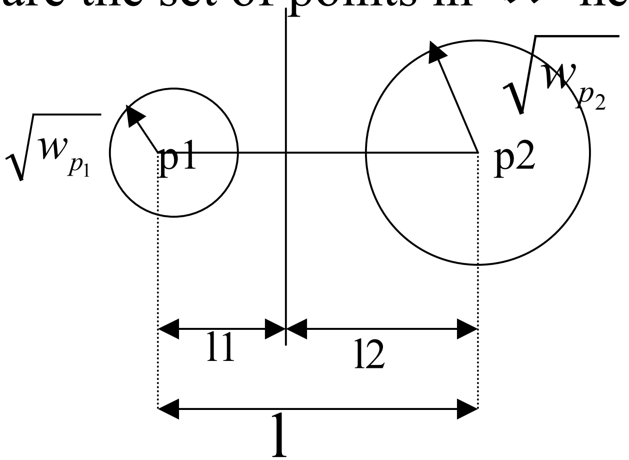
Power Diagram (PD) of a weighted point set

Tiling of space into convex regions where ith region (tile) are the set of points in $\Re^d$ nearest to p_i in the power distance metric.



$$\pi_{p_1} = l_1^{\ 2} - w_{p_1} = l_2^{\ 2} - w_{p_2} = \pi_{p_2}$$

Bisector Plane which matches power distance.

Regular Triangulation (RT)

Dual of Power Diagram (PD) with an edge of RT for each Bisector Plane of PD

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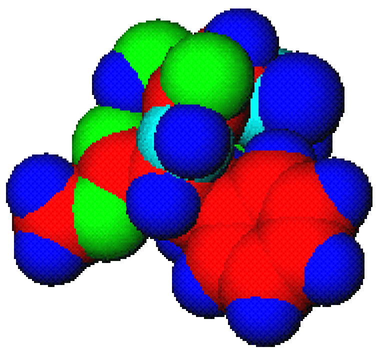
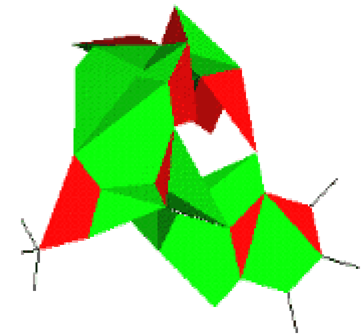
Particle Data to Meshes

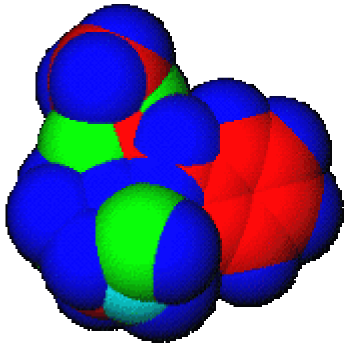
Atomic Centers



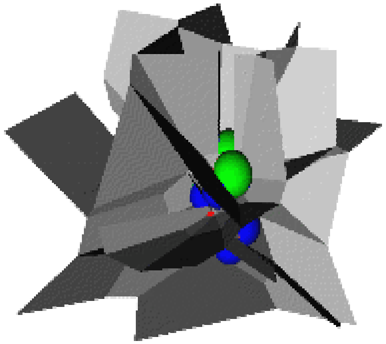
CPK



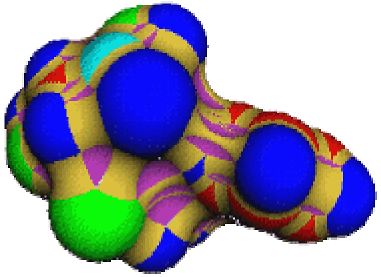
CPK Alpha-Shape



Solvent Accessible Surface (SAS)



Power Diagram of SAS



Solvent Excluded Surface (SES)

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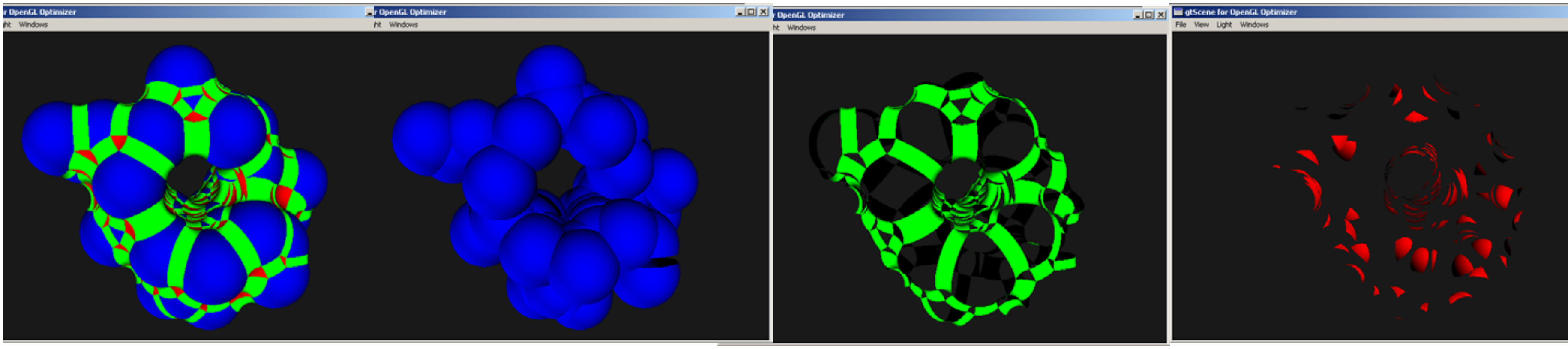
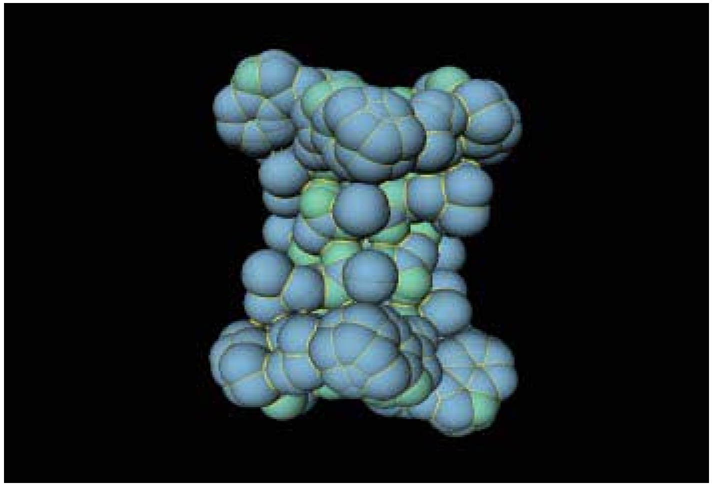
Molecular Surfaces (Solvent Excluded Surface)



SES = spherical patches + toroidal patches + concave patches



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Field Data

- Scalar

  temperature, pressure, density, energy, change, resistance, capacitance, refractive index, wavelength, frequency & fluid content.

- Vector

  velocity, acceleration, angular velocity, force, momentum, magnetic field, electric field, gravitational field, current, surface normal

- Tensor

  stress, strain, conductivity, moment of inertia and electromagnetic field

- Multivariate Time Series

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Field Data on Meshes

- Finite elements commonly used
  - Linear finite elements
  - Non-linear finite elements
- Interpolants/Approximants

  used to approximate the data on the domain

(Lagrange, Hermite, …)

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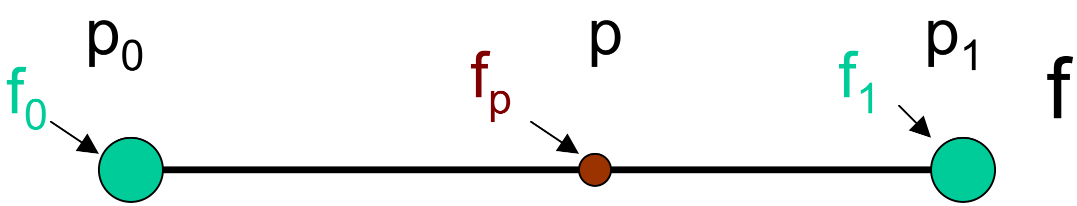
Linear Interpolation on a line segment



The Barycentric coordinates $\alpha = (\alpha_0\ \alpha_1)$ for any point p on line segment $<p_0\ p_1>$, are given by

$$\alpha = \left(\frac{dist(p, p_1)}{dist(p_0, p_1)}, \frac{dist(p_0, p)}{dist(p_0, p_1)}\right)$$

which yields $\boxed{p = \alpha_0\ p_0 + \alpha_1\ p_1}$

and $\boxed{f_p = \alpha_0\ f_0 + \alpha_1\ f_1}$

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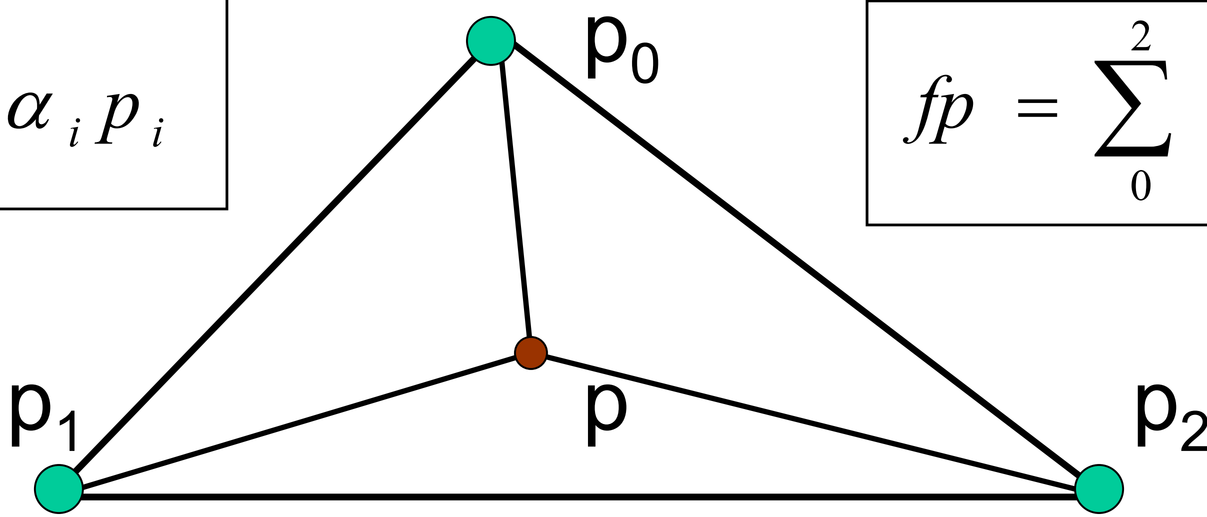
Linear interpolation over a triangle

$$p = \sum_{0}^{2} \alpha_i p_i$$



$$fp = \sum_{0}^{2} \alpha_i fp_i$$

For a triangle p_0,p_1,p_2, the Barycentric coordinates

$\alpha = (\alpha_0\, \alpha_1\, \alpha_2)$ for point p,

$$\alpha = \left(\frac{area(p,p_1,p_2)}{area(p_0,p_1,p_2)}, \frac{area(p_0,p,p_2)}{area(p_0,p_1,p_2)}, \frac{area(p_0,p_1,p)}{area(p_0,p_1,p_2)}\right)$$

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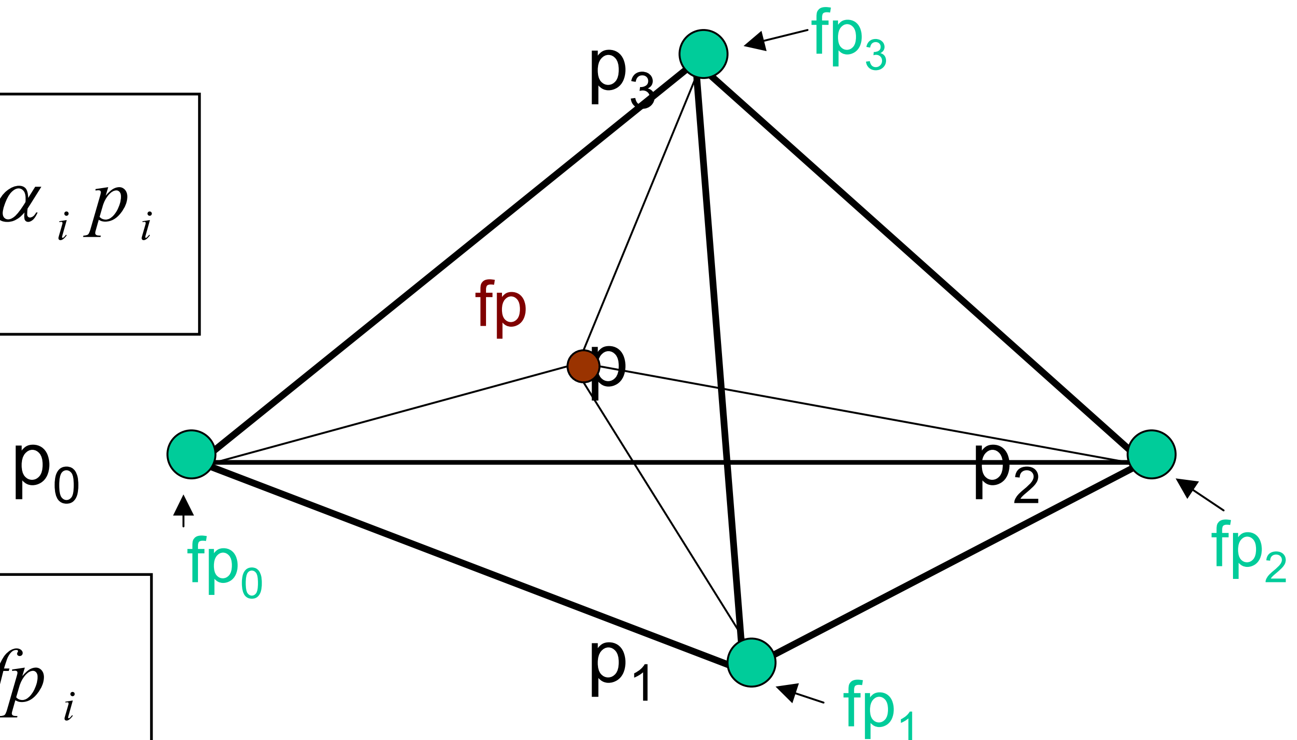
Linear interpolant over a tetrahedron

Linear Interpolation within a

- Tetrahedron (p_0,p_1,p_2,p_3)

  $\alpha = \alpha_i$ are the barycentric coordinates of p

$$p = \sum_{0}^{3} \alpha_i p_i$$



$$fp = \sum_{0}^{3} \alpha_i fp_i$$

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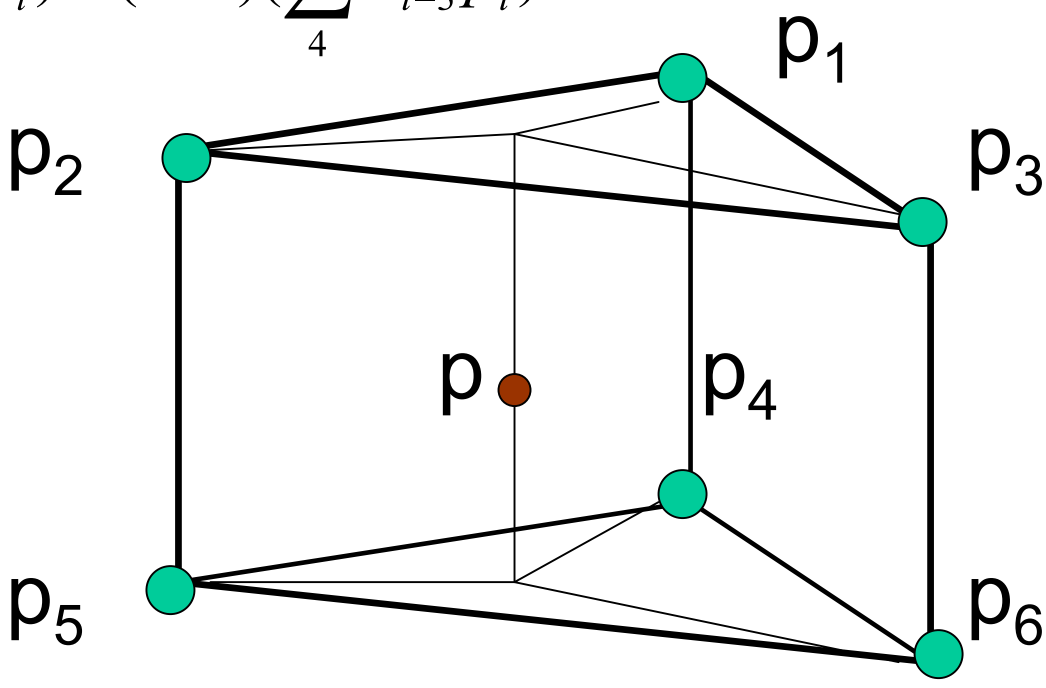
Other 3D Finite Elements (contd)

- Unit Prism (p_1,p_2,p_3,p_4,p_5,p_6)

$$p = t\left(\sum_{1}^{3} \alpha_i p_i\right) + (1-t)\left(\sum_{4}^{6} \alpha_{i-3} p_i\right)$$

Note: nonlinear



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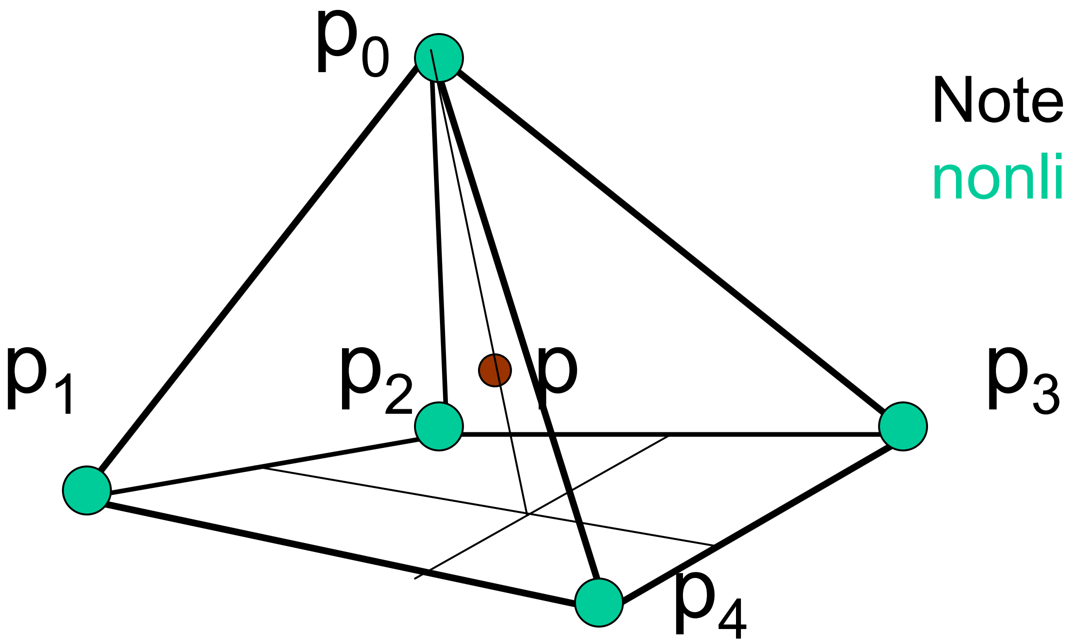
Other 3D Finite elements

- Unit Pyramid (p_0, p_1, p_2, p_3, p_4)

$$p = up_0 + (1-u)(t(sp_1 + (1-s)p_2) + (1-t)(sp_3 + (1-s)p_4))$$



Note:
nonlinear

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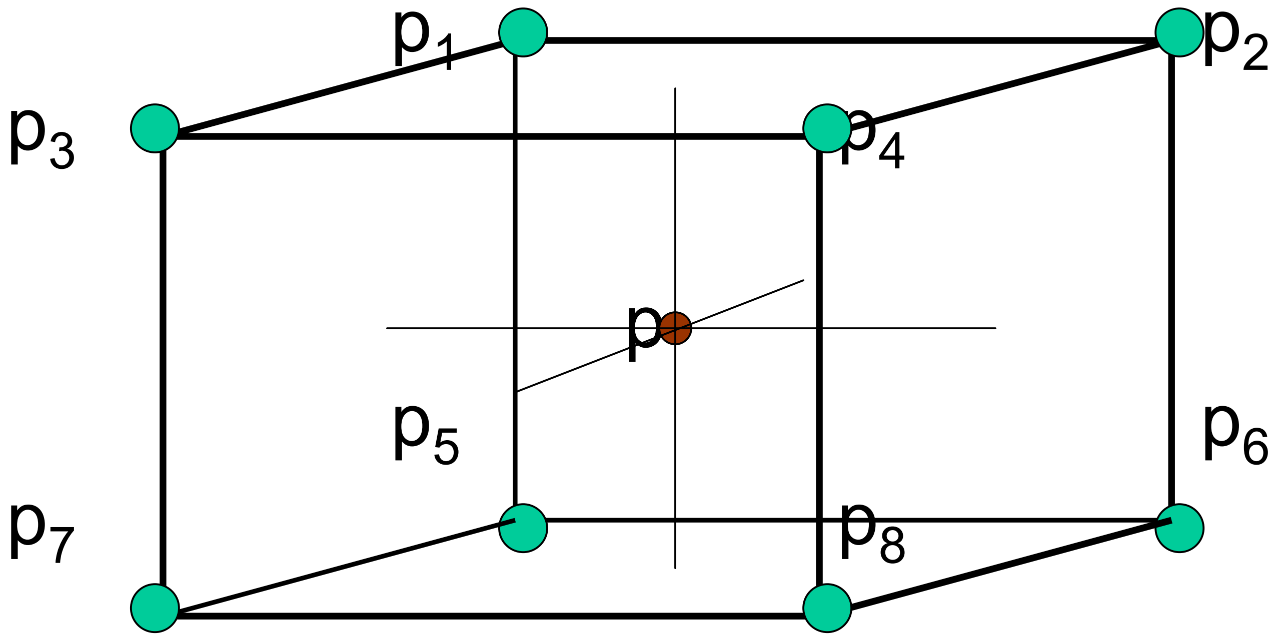
Other 3D Finite Elements

- Unit Cube ($p_1,p_2,p_3,p_4,p_5,p_6,p_7,p_8$)
  - Tensor in all 3 dimensions

$$p = u(t(sp_1 + (1-s)p_2) + (1-t)(sp_3 + (1-s)p_4)) +$$
$$(1-u)(t(sp_5 + (1-s)p_6) + (1-t)(sp_7 + (1-s)p_8))$$

Trilinear interpolant



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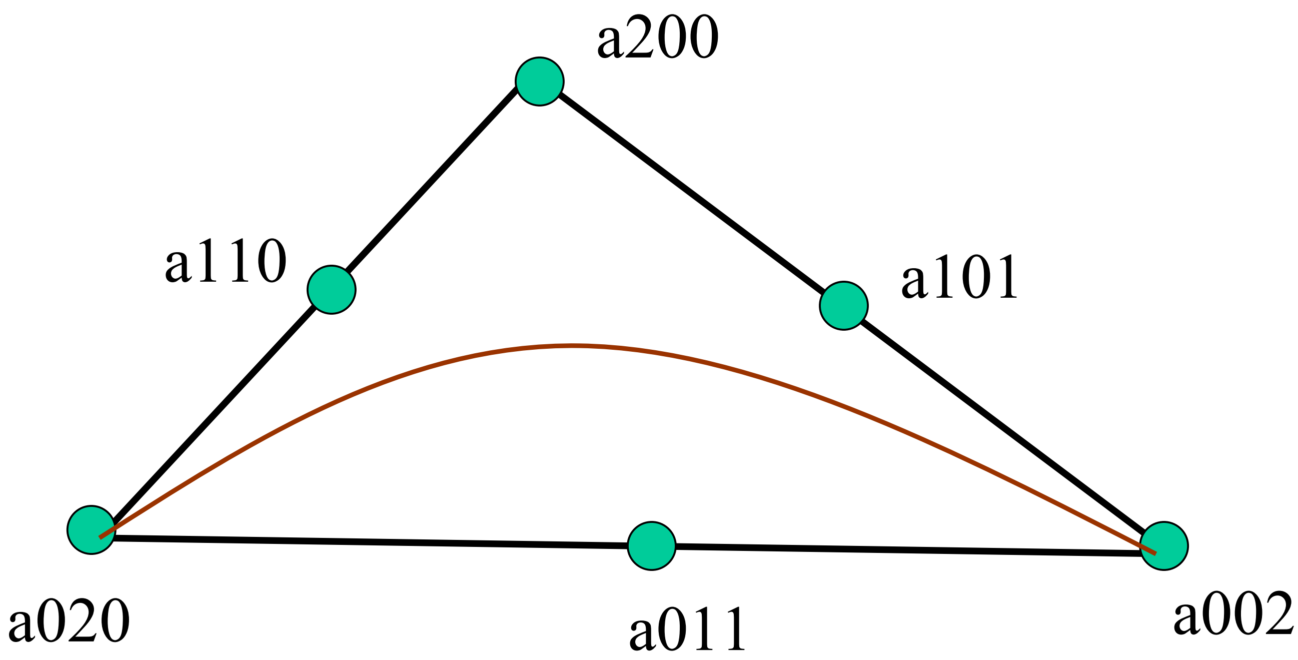
Can we construct Good Non-Linear Curve and Surface Finite Elements ?



The conic curve interpolant is the zero of the bivariate quadratic polynomial interpolant over the triangle

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Every good answer needs coffee! Or Mineralwasser !!

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Non-Linear Representations

- Explicit
  - Curve: $y = f(x)$
  - Surface: $z = f(x,y)$
  - Volume: $w = f(x,y,z)$
- Implicit
  - Curve: $f(x,y) = 0$ in 2D, $<f_1(x,y,z) = f_2(x,y,z) = 0>$ in 3D
  - Surface: $f(x,y,z) = 0$
  - Interval Volume: $c_1 < f(x,y,z) < c_2$
- Parametric
  - Curve: $x = f_1(t)$, $y = f_2(t)$
  - Surface: $x = f_1(s,t)$, $y = f_2(s,t)$, $z = f_3(s,t)$

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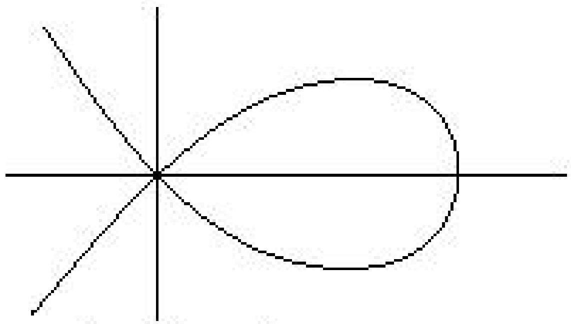
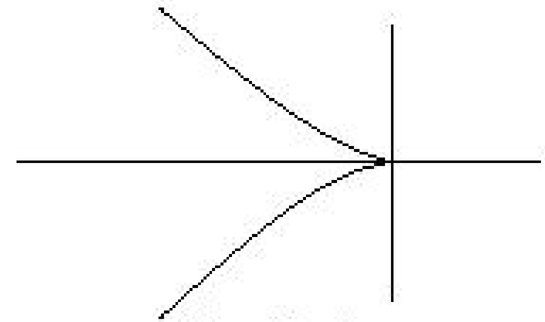
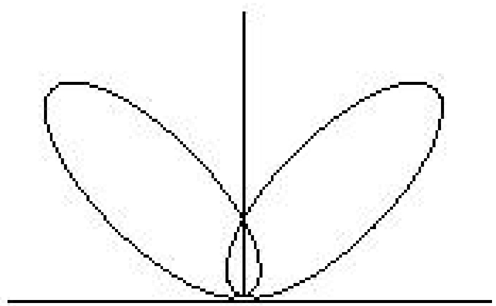
Algebraic Curves



$x^3 - x^2 + y^2 = 0$



$y^3 - x^2 = 0$



$2x^4-3x^2y+y^2-2y^3+y^4 = 0$

$x^4+x^2y^2-2x^2y-xy^2+y^2 = 0$

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Implict vs Parametric

- Curves
- Surfaces
- Volumes

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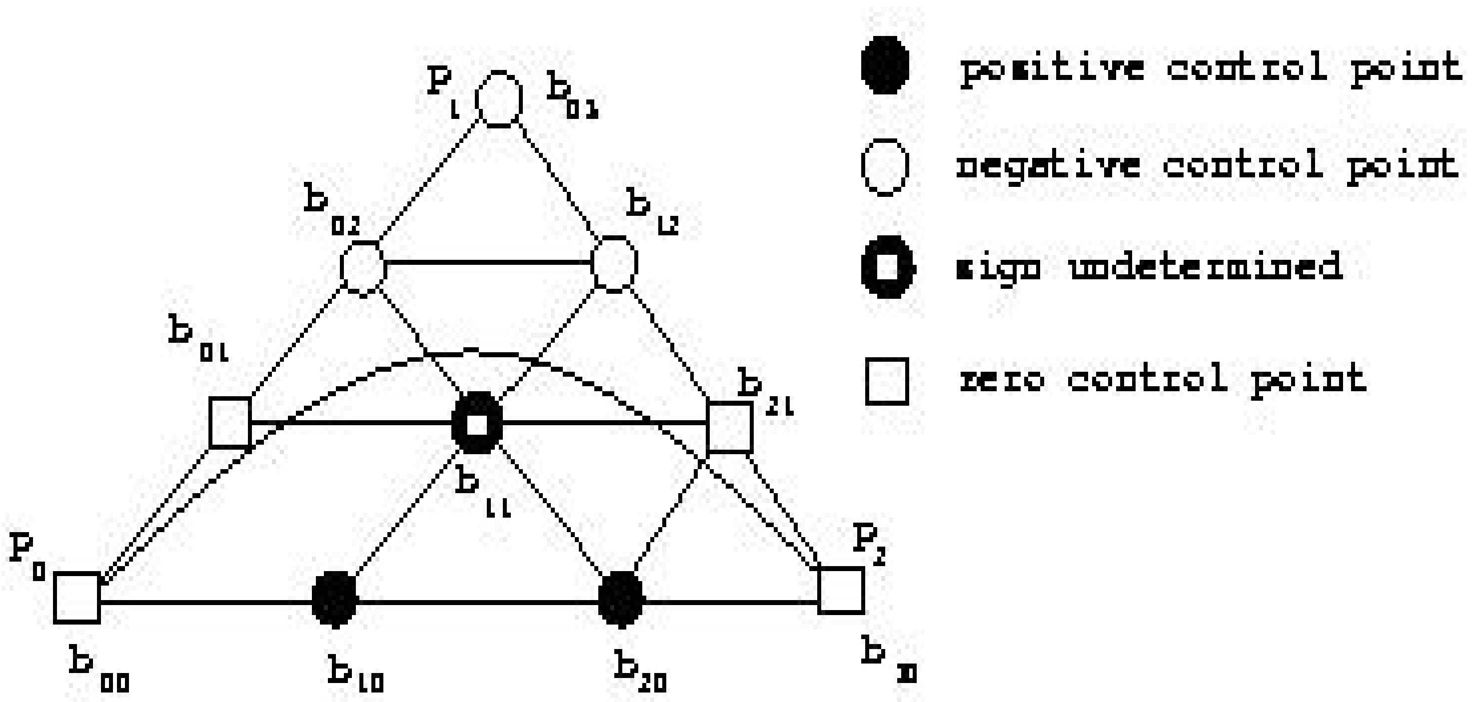
A-spline segment over BB basis



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Discriminating Curve Family

For a given triangle or quadrilateral R, let R_1 and R_2 be two closed boundaries of R and let $D = \{A_s(x,y) = \gamma(x,y) - s\,\delta(x,y) = 0 : s \in [0,1]\}$ be an algebraic curve family with s as a parameter and $\delta(x,y) > 0$ on $R \setminus \{R_1, R_2\}$ such that

1. $R_1 \cap R_2 = \varnothing$.
2. Each curve in D passes through R_1 and R_2.
3. Each curve in D is regular in the interior of R.
4. For $\forall p \in R \setminus \{R_1, R_2\}$, there exists one and only one $s \in [0, 1]$ such that $A_s(p) = 0$.

Then we say D is a discriminating family on R, denoted by $D(R, R_1, R_2)$.

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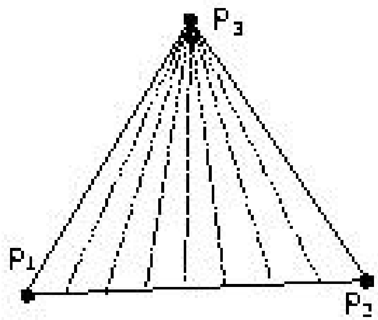
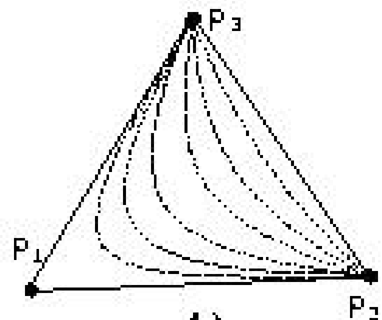
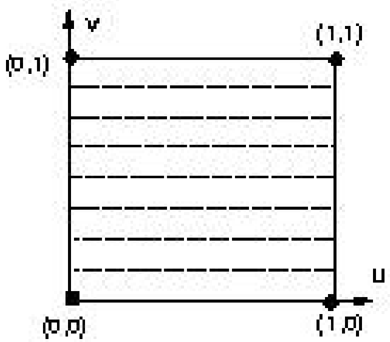
Examples of Discriminating Curve Families



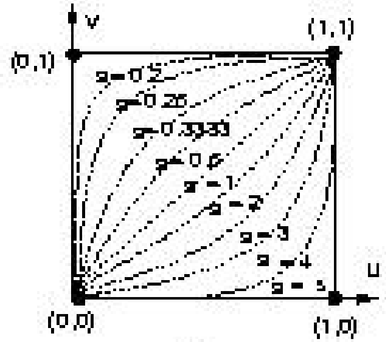
(a)



(b)



(a)



(b)

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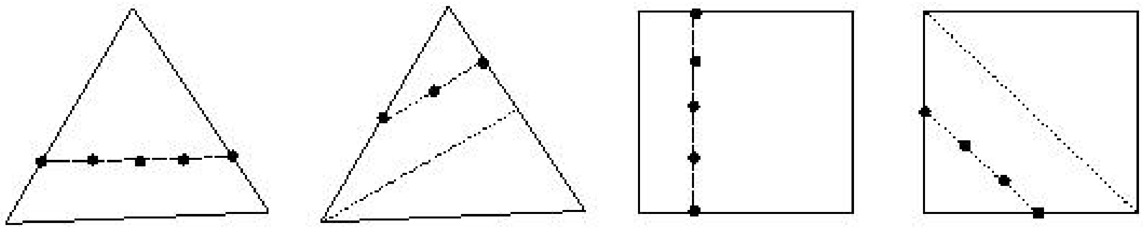
A-spline Segment



(a) (b) (c) (d)

For a given discriminating family D(R, R_1, R_2), let f(x, y) be a bivariate polynomial . If the curve f(x, y) = 0 intersects with each curve in D(R, R_1, R_2) only once in the interior of R, we say the curve f = 0 is regular(or A-spline segment) with respect to D(R, R_1, R_2).

If $B_0(s)$, $B_1(s)$, … have one sign change, then the curve is

(a) D_1 - regular curve.

(b) D_2 - regular curve.

(c) D_3 - regular curve.

(d) D_4 - regular curve.

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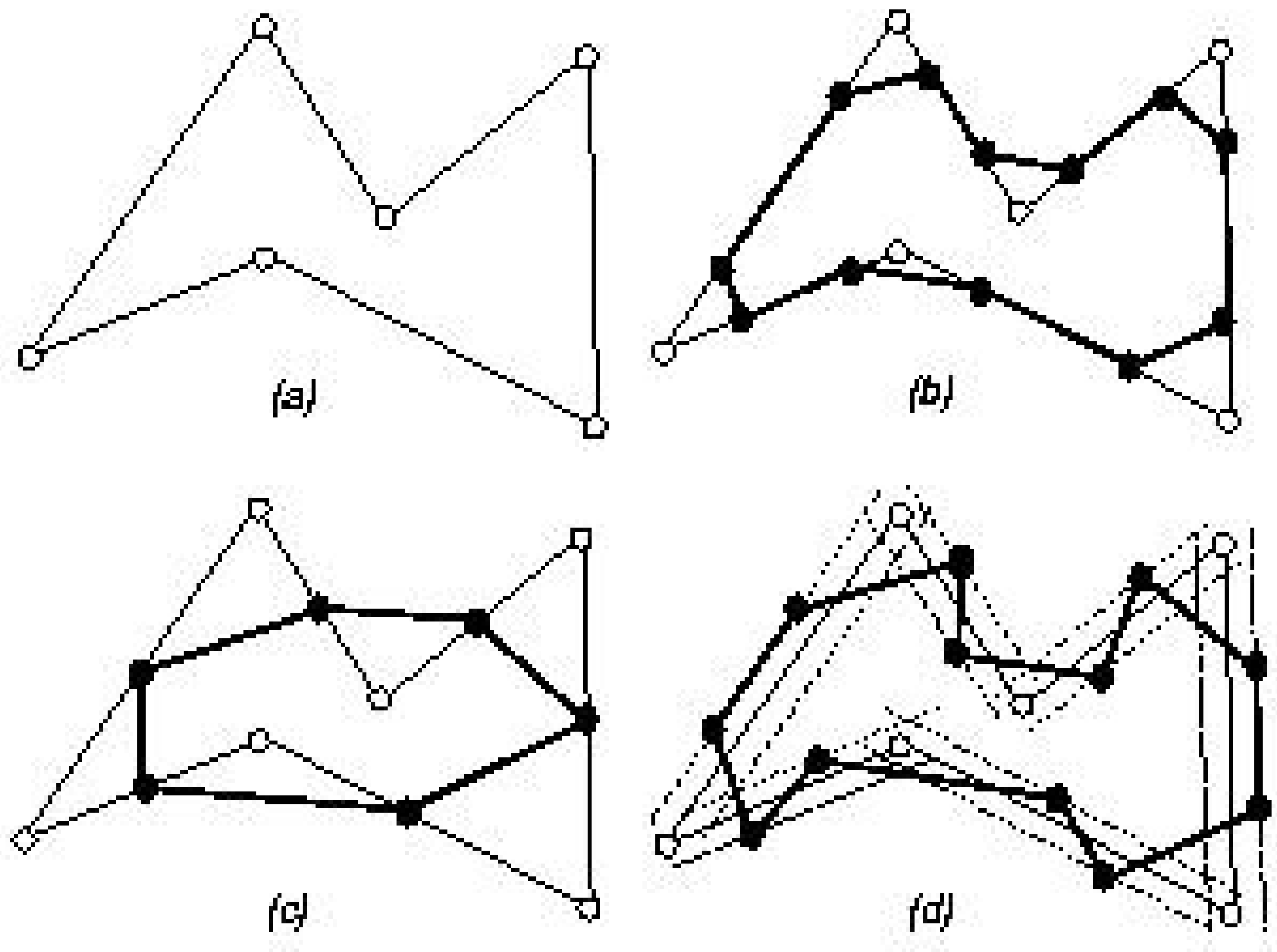
Constructing Scaffolds



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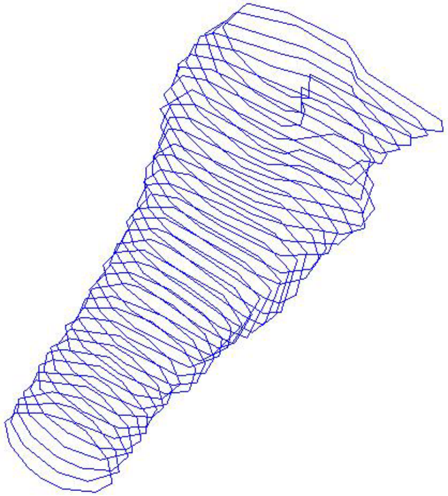
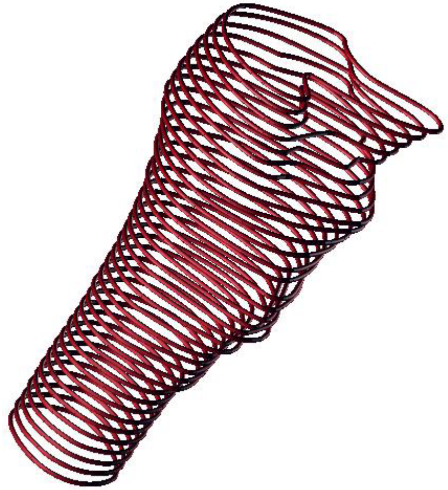
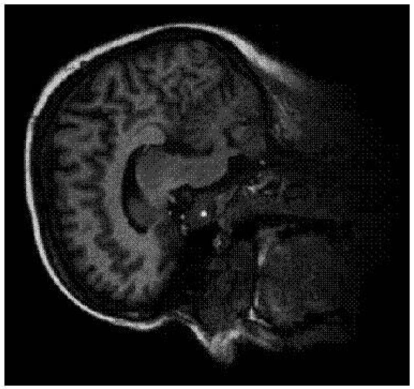
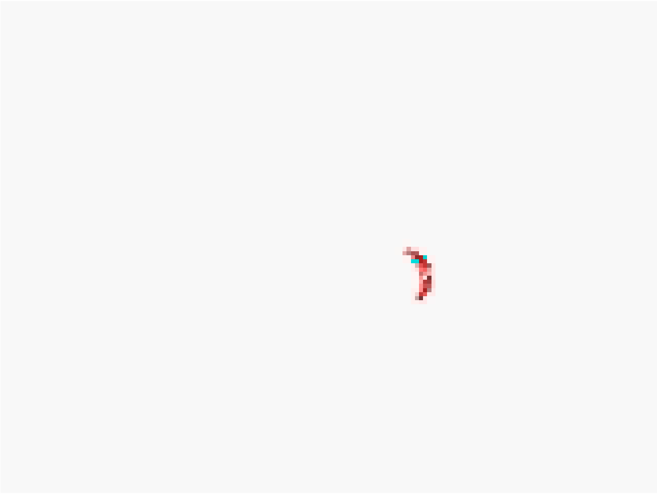
C^1 A-spline Reconstructions









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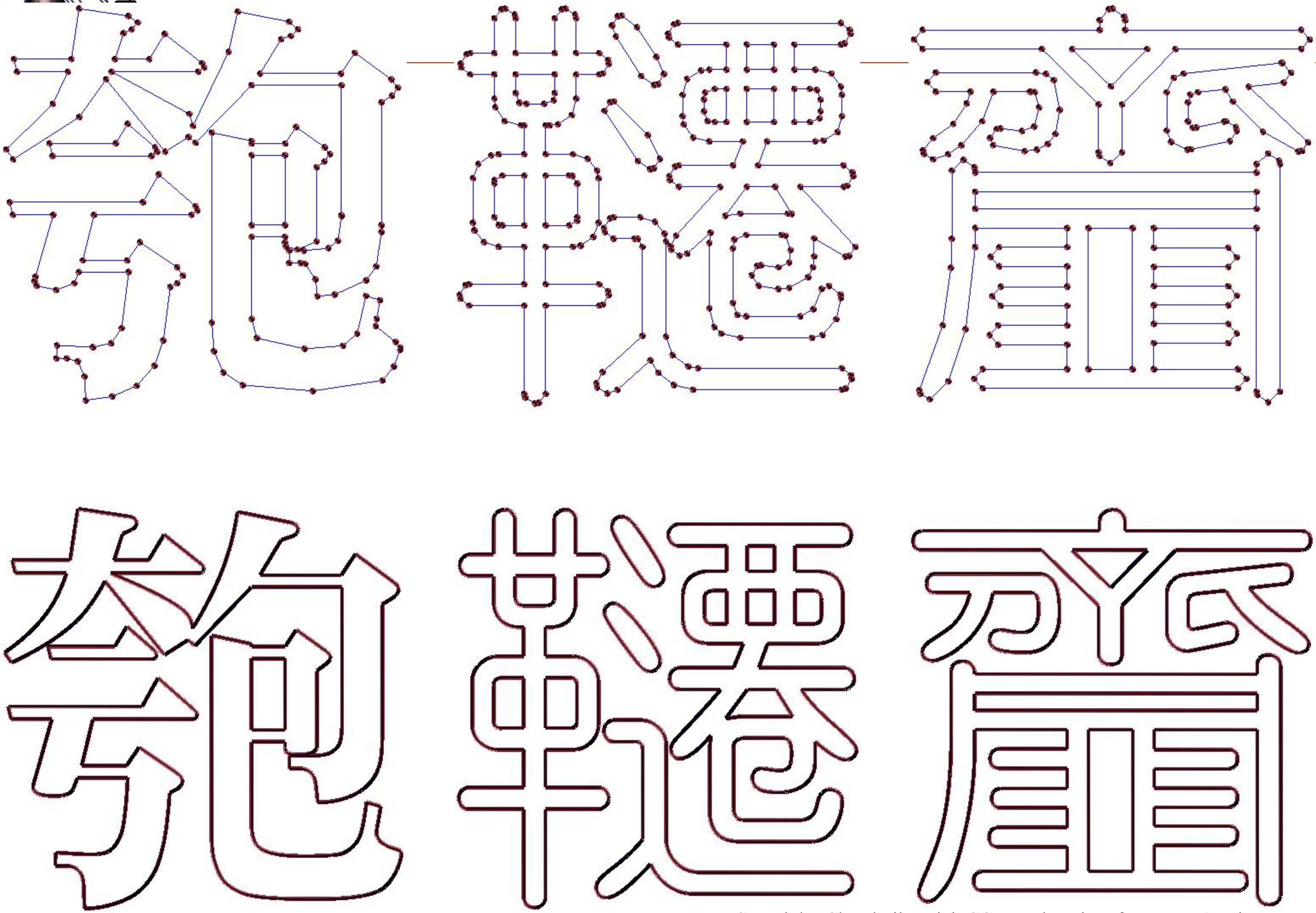
Input:



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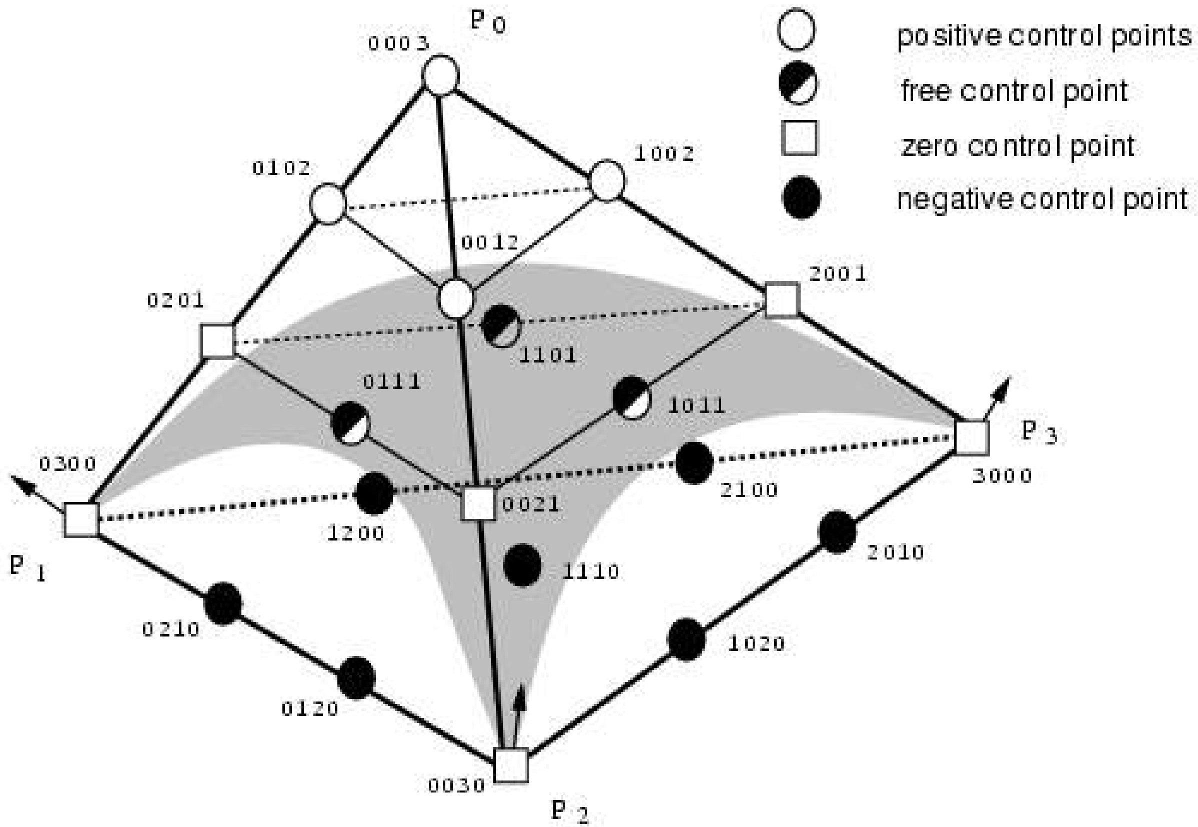
A-patch Surface (C^1) Interpolant



- An implicit single-sheeted interpolant over a tetrahedron

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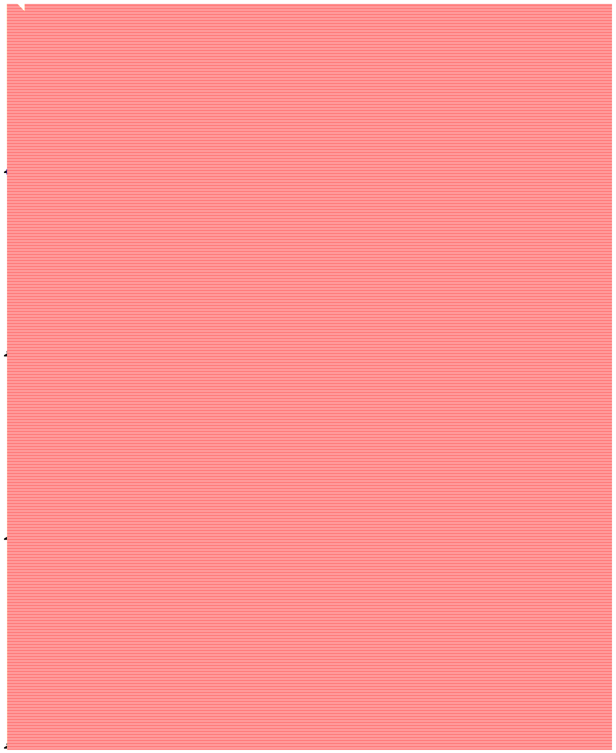
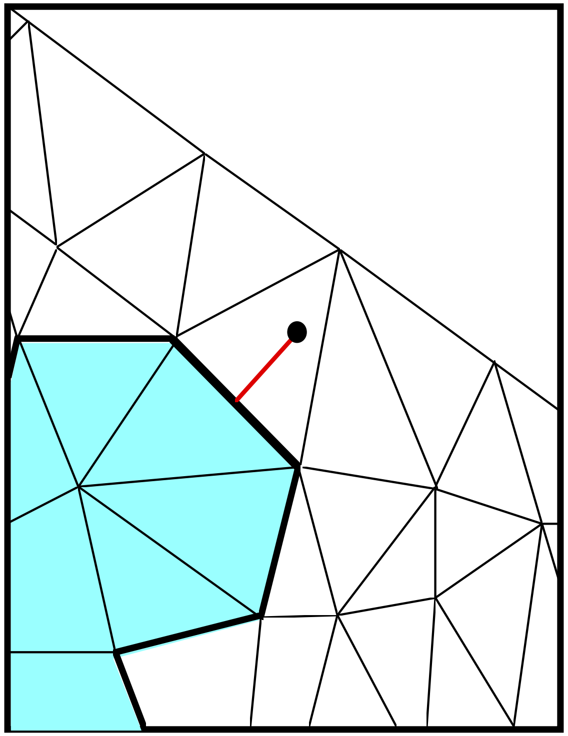
Incremental Scaffolding and Function Construction

- **DAG**



Signed distance



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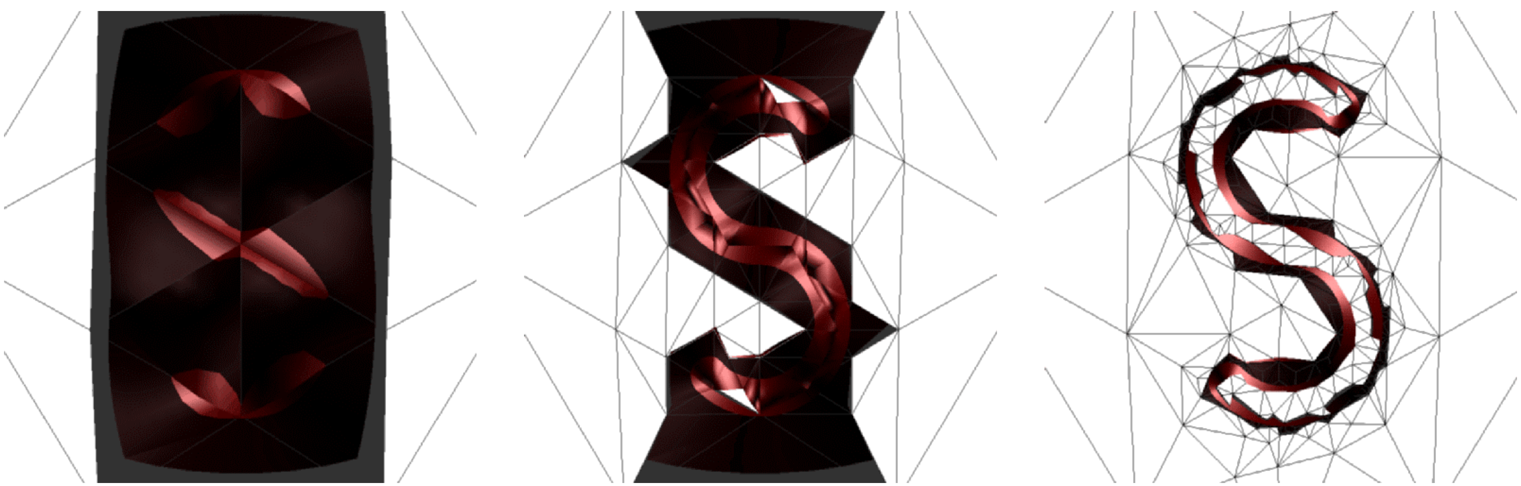
Incremental refinement

Bivariate Case



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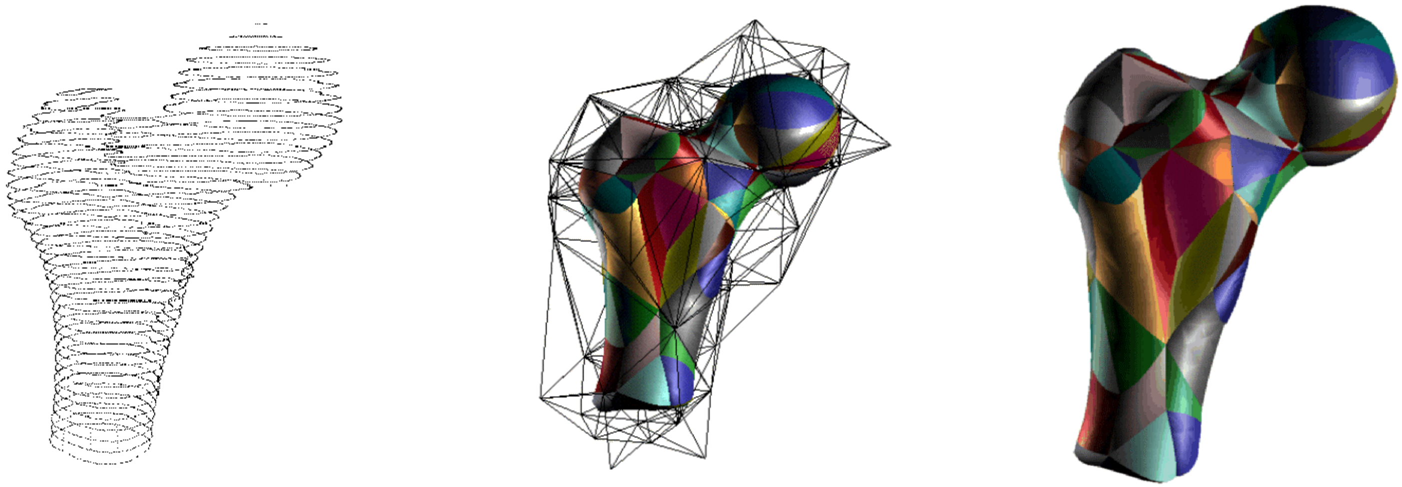
A-patch surface models

- ~9200 points, 406 patches (degree 3), 1% error



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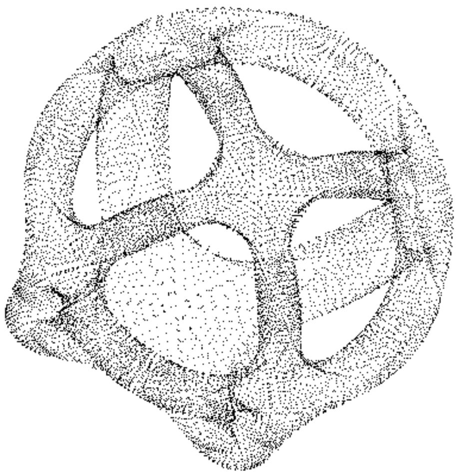
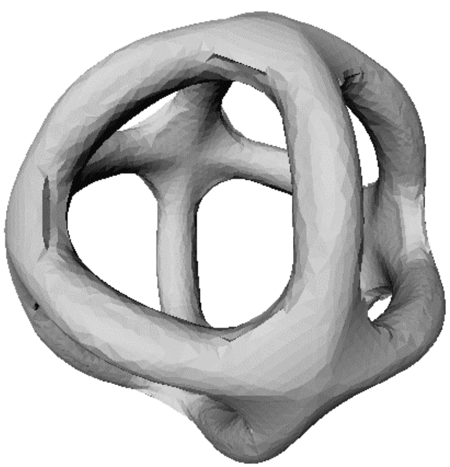
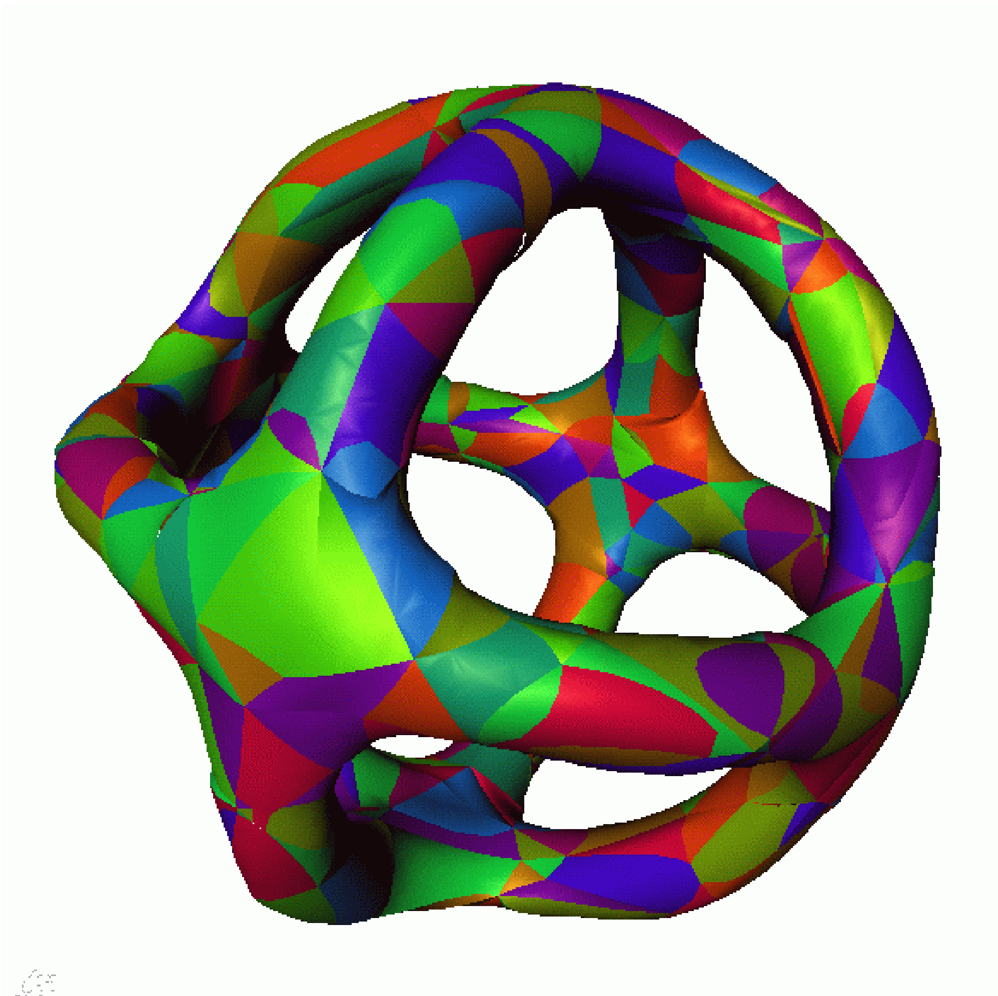
High Genus Example







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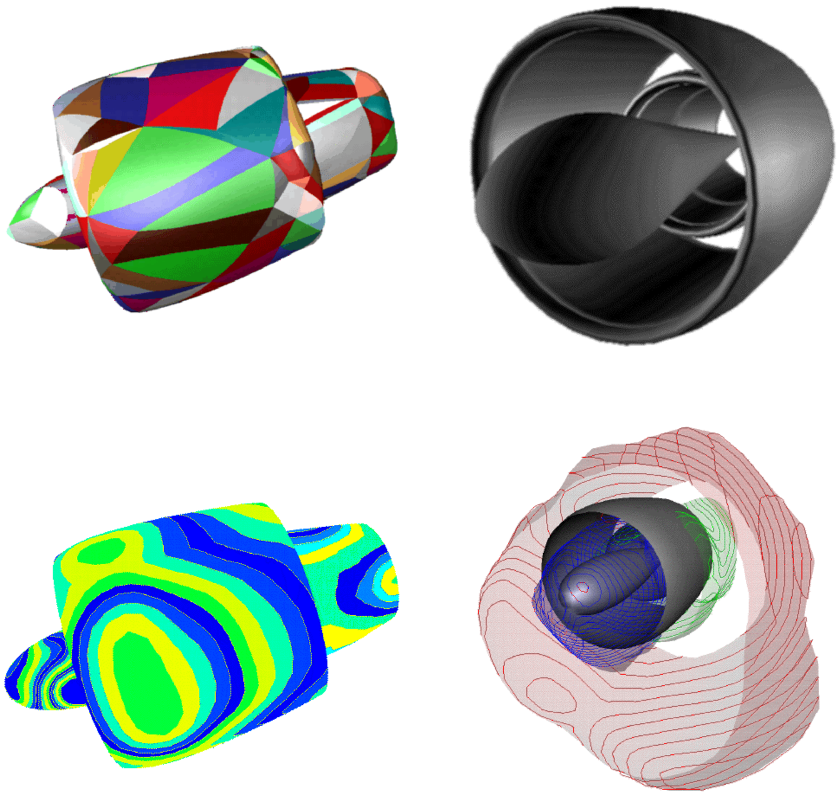
Results

- ~10^4 points, 460 patches (degree 3), 1% error



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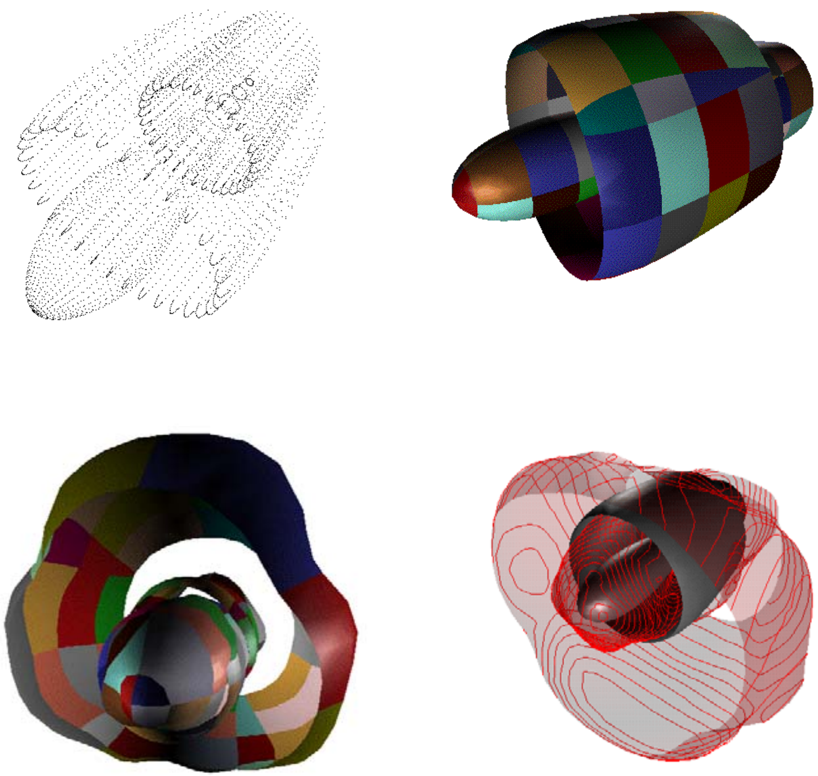
Tensor-product patches
Results

- ~10^4 points,
  180 patches
  (degree 3),
  1% error



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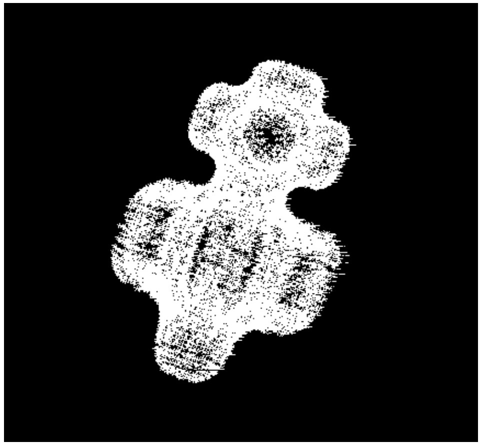
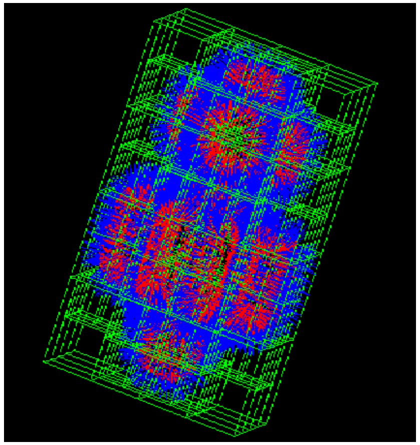
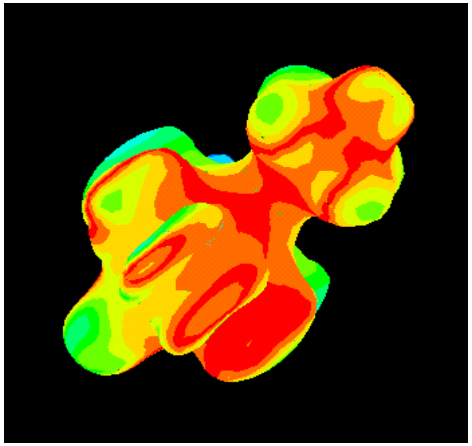
Tensor-product patches
Manifold and Function Data







Data points **Final Mesh** **Implicit patches** **Function**

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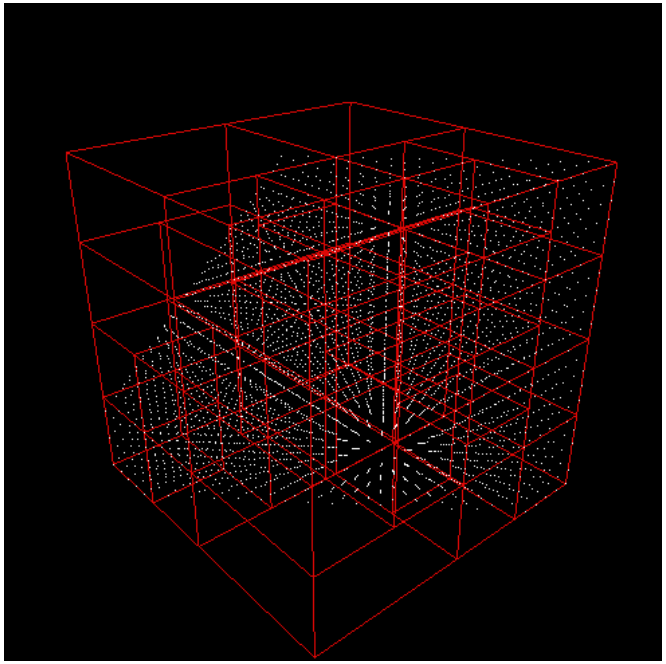
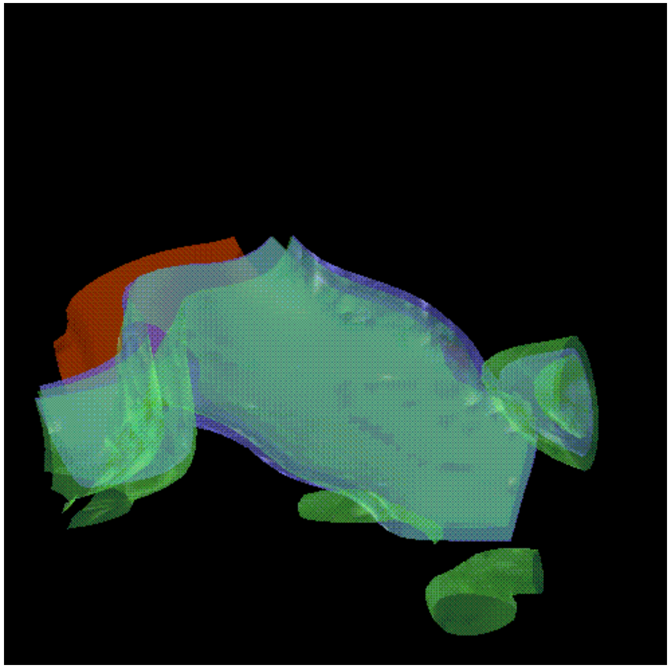
Tensor-product A-patches Volumetric Data

~10^4 points, 220 patches (degree 3), 1% error





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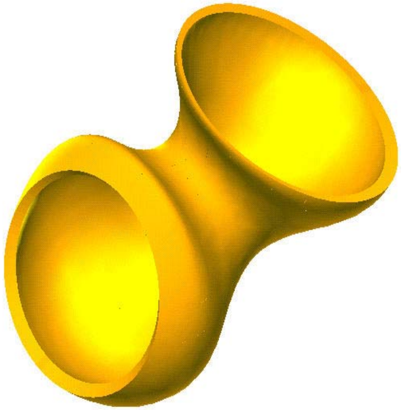
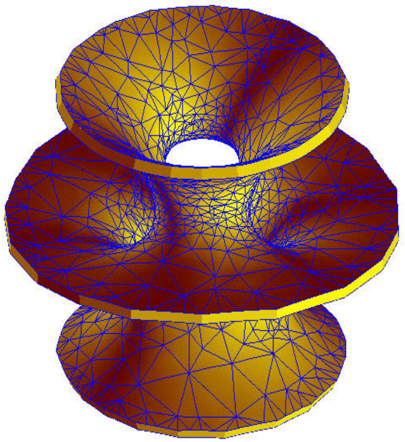
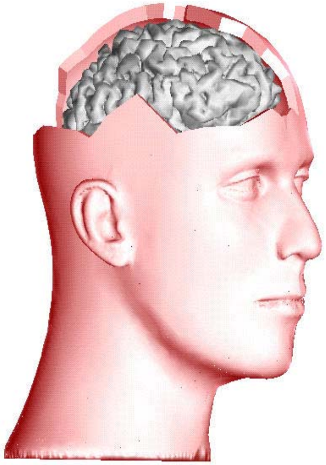
Shell Finite Elements

- Airfoils
- Tin cans
- Shell canisters
- Sea shells
- Earth's outer crust
- Human skin
- Skeletal Structures

• • •







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C^1 Interpolant

Hermite interpolation

$$f(t) = f_0 H_0^3(t) + f_0' H_1^3(t) + f_1 H_2^3(t) + f_1' H_3^3(t)$$

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Incremental Basis Construction

- Define functions and gradients on the edges of a prism
- Define functions and gradients on the faces of a prism
- Define functions on a volume
- Blending

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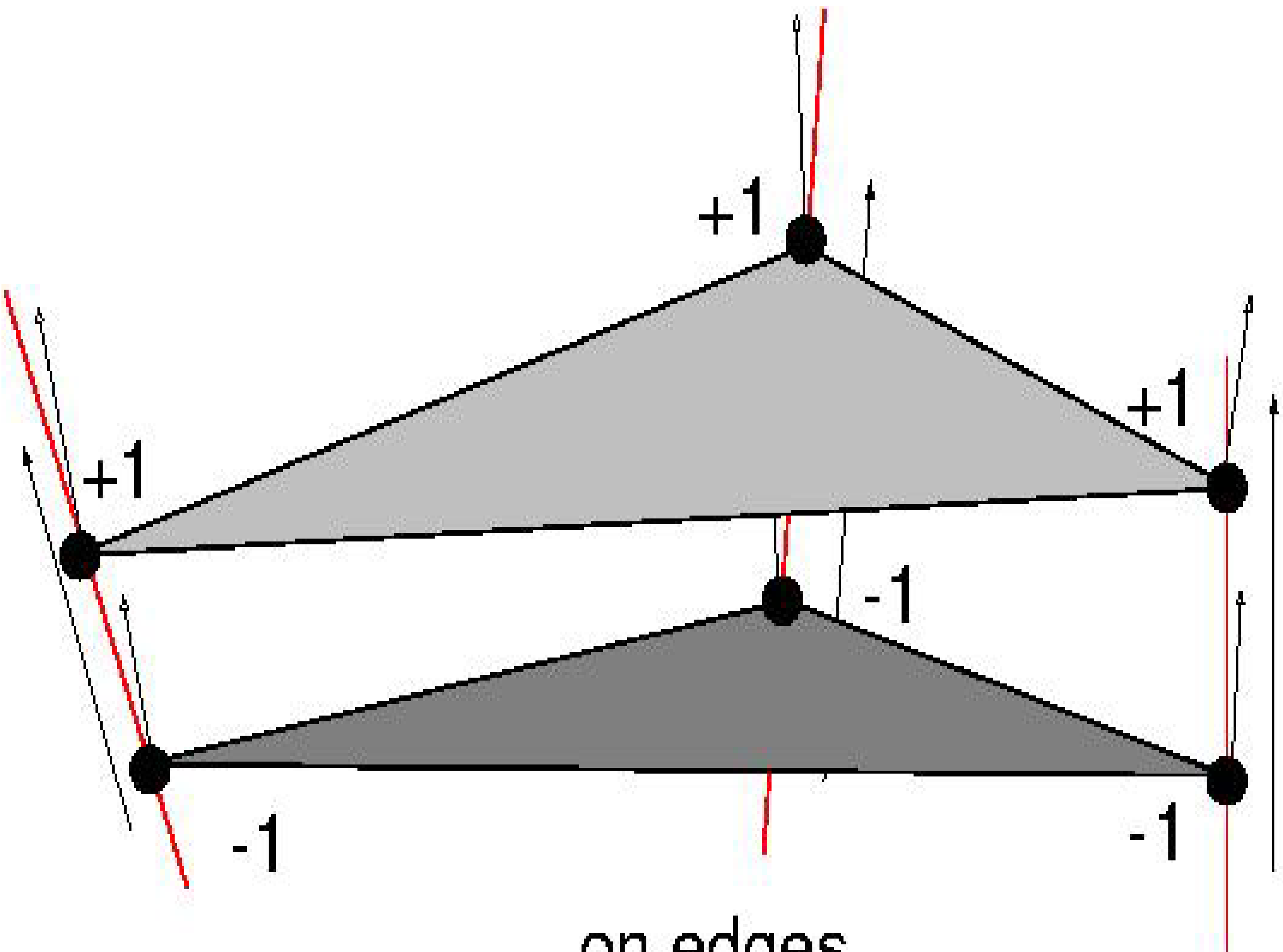
Hermite Interpolant on Prism Edges



on edges

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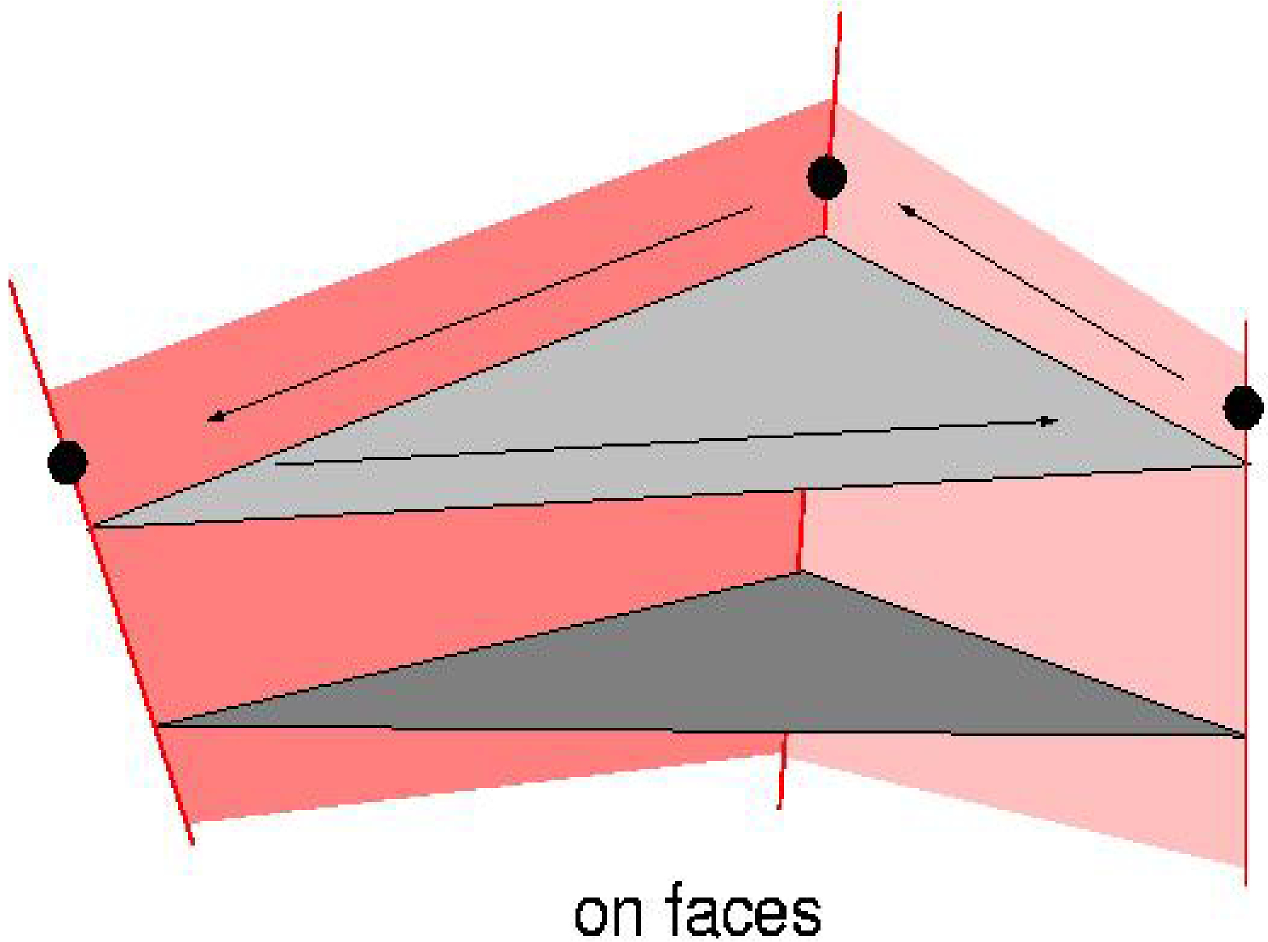
Hermite Interpolation on Prism Faces



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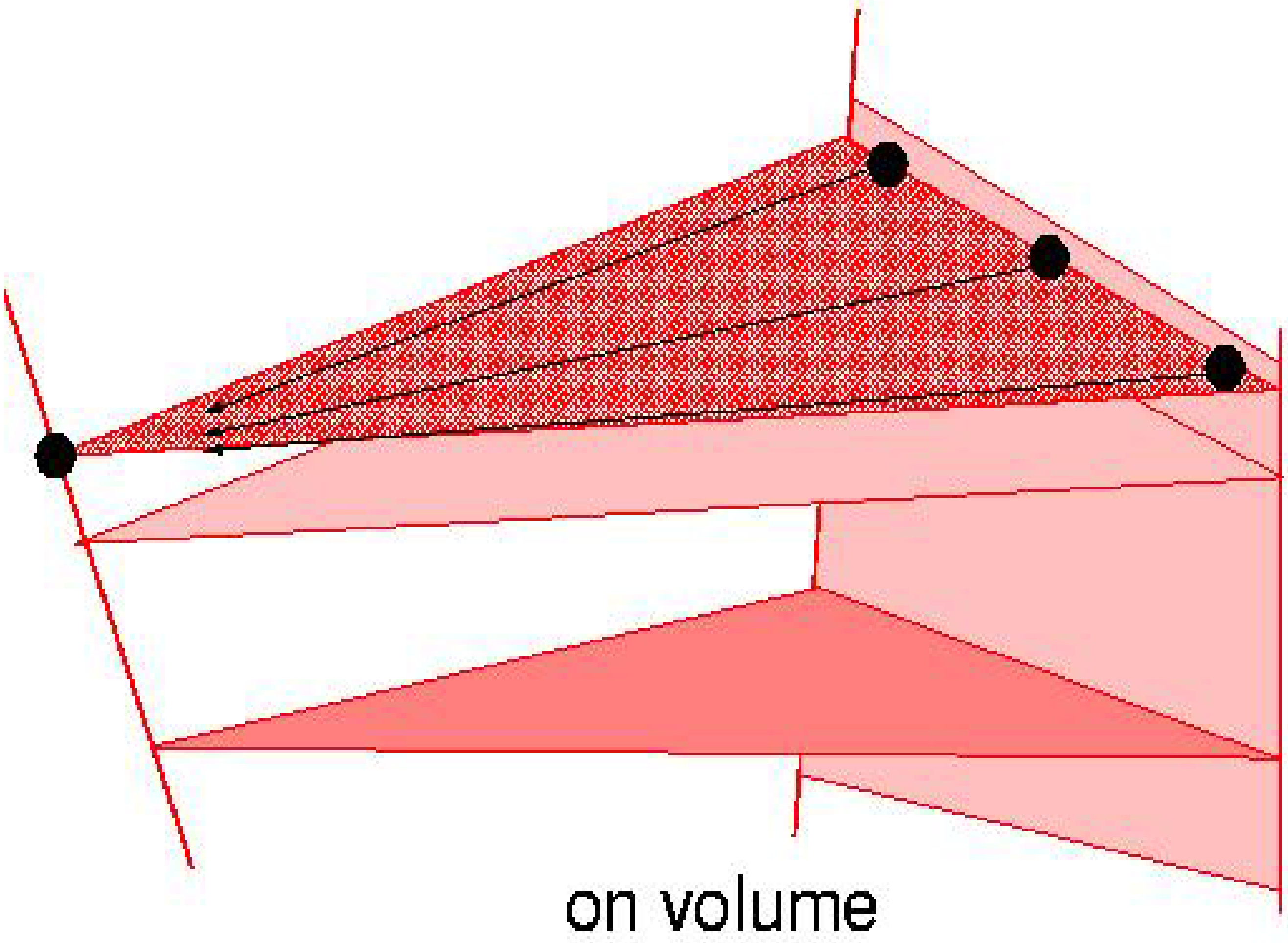
Side Vertex Interpolation



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C^1 function construction (cont.)

- Blending

$$\sum W_i D_i(b_1, b_2, b_3, \lambda) + (b_1 b_2 b_3)^2 E(b_1, b_2, b_3, \lambda)$$

where

$$W_i(b_1, b_2, b_3) = \frac{(b_j b_k)^\beta}{(b_2 b_3)^\beta + (b_1 b_3)^\beta + (b_1 b_2)^\beta}, \beta > 1$$

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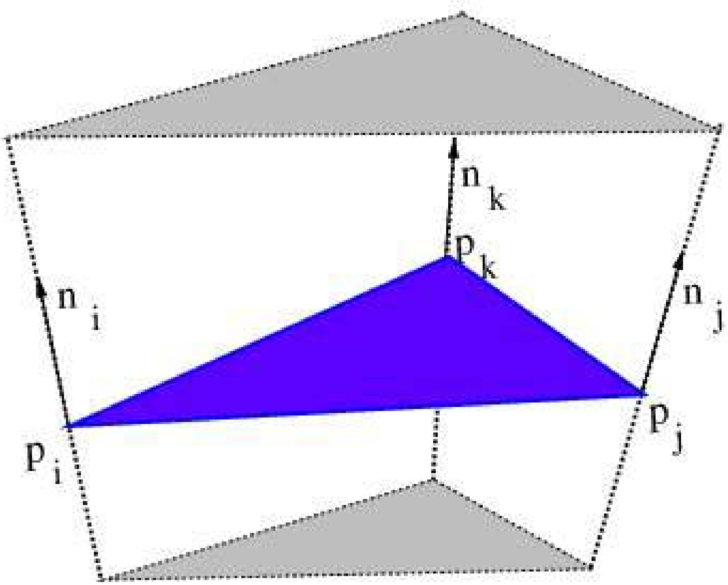
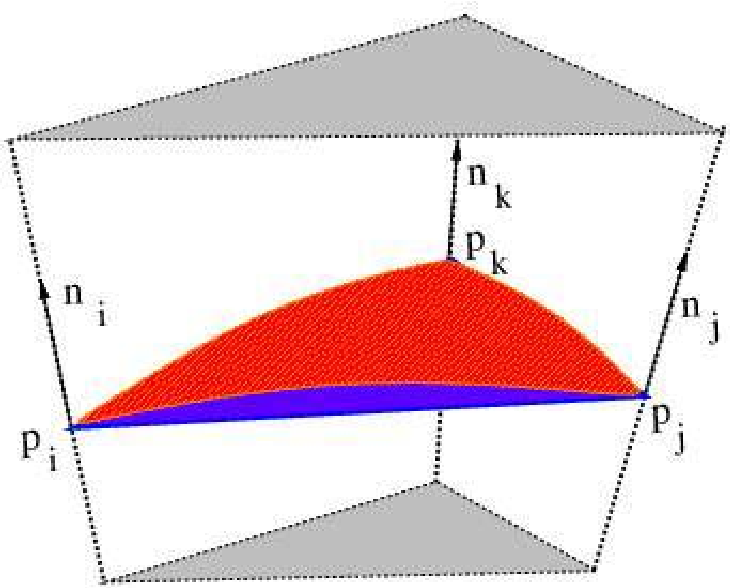
Shell Elements (contd)





- The function F is C^1 over Σ and interpolates C^1 (Hermite) data
- The interpolant has quadratic precision

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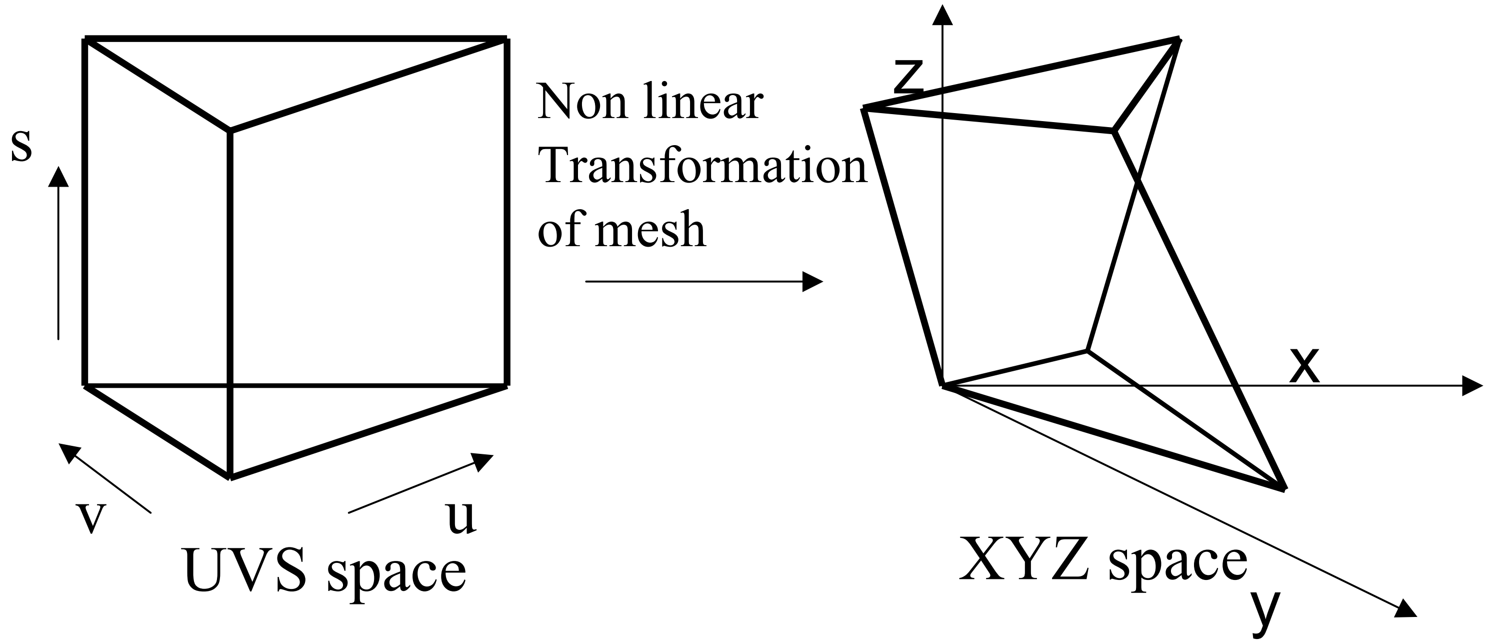
Non-linear finite elements-3d

- Irregular prism
  - Irregular prisms have been used to represent data.



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = ((1-u-v)\overrightarrow{\mathbf{p_0}} + u\overrightarrow{\mathbf{p_1}} + v\overrightarrow{\mathbf{p_2}})(1-s) + ((1-u-v)\overrightarrow{\mathbf{p_3}} + u\overrightarrow{\mathbf{p_4}} + v\overrightarrow{\mathbf{p_5}})(s)$$

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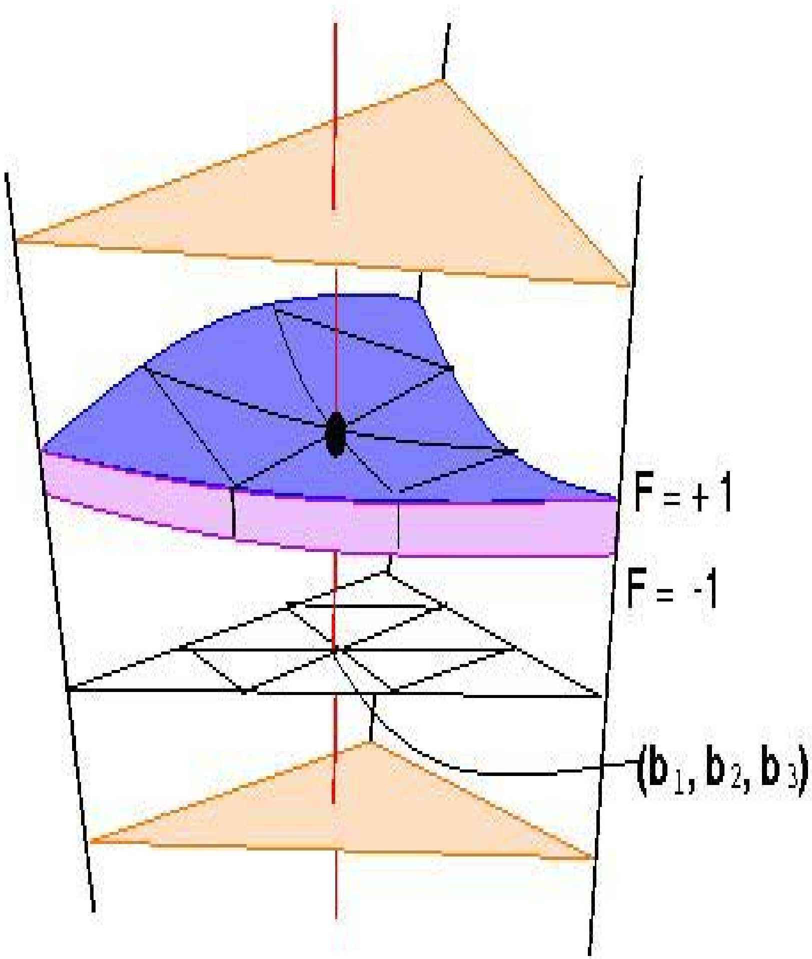
Evaluation



- For each (b_1, b_1, b_3),
- $b_i \geq 0, \sum b_i = 1,$ Find the intersection of F=α and the line

$$b_1 v_i(\lambda) + b_2 v_j(\lambda) + b_3 v_k(\lambda)$$

That is find the zero

$$\phi(\lambda) = F(b_1, b_2, b_3, \lambda) = \alpha$$

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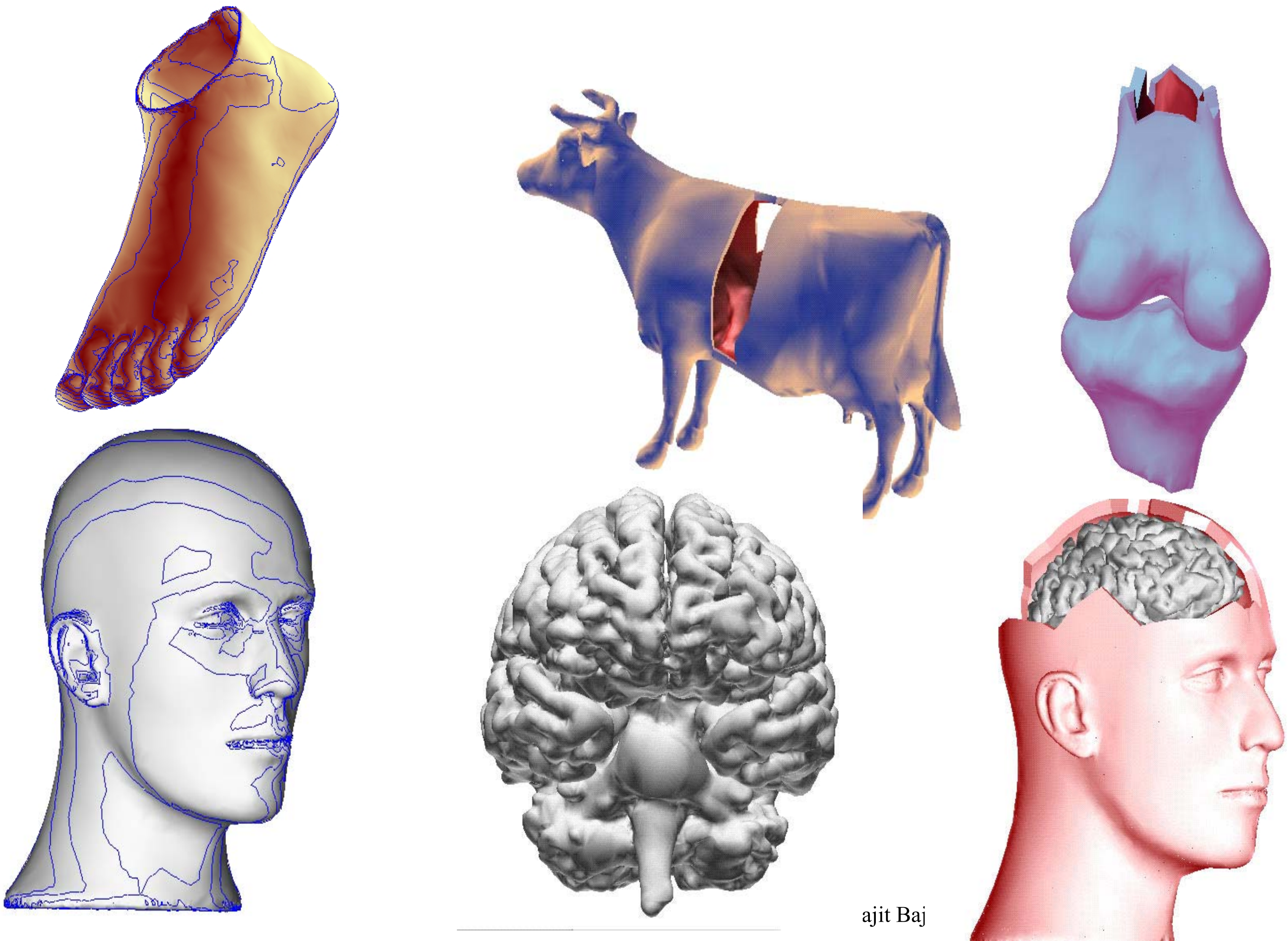
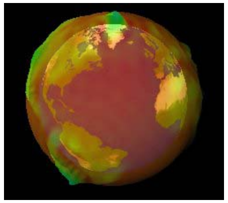
Examples with Shell Finite Elements

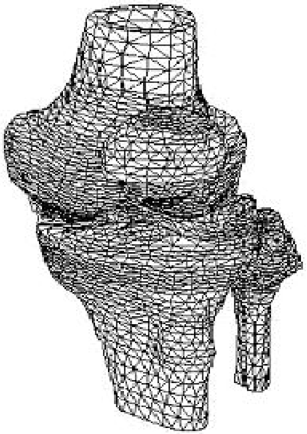




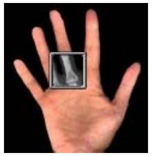
Computational Visualization

1. Sources, characteristics, representation

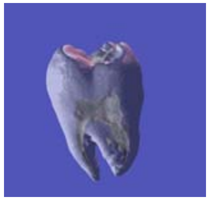
2. Mesh Processing



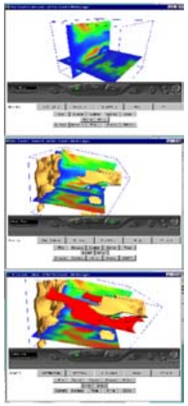
3. Contouring



4. Volume Rendering



5. Flow, Vector, Tensor Field Visualization



6. Application Case Studies

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Further Reading

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- **C^1 Modeling with A-patches from Rational Trivariate Functions,** *Computer Aided Geometric Design, 18:3(2001), 221-243*

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- **Smooth Shell Construction with Mixed Prism Fat Surfaces,** C.Bajaj, G. Xu, Geometric Modeling, Springer Verlag, Computing Supplementum 14, 2001, pg 19 - 36
- **Implicit Surface Patches**, C. Bajaj, Introduction to Implicit Surfaces, edited by J. Bloomenthal, Morgan Kaufman Publishers, (1997), 98 – 125
- **Automatic Reconstruction of Surfaces and Scalar Fields from 3D Scans**
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- **Modeling Physical Fields for Interrogative Data Visualization,**
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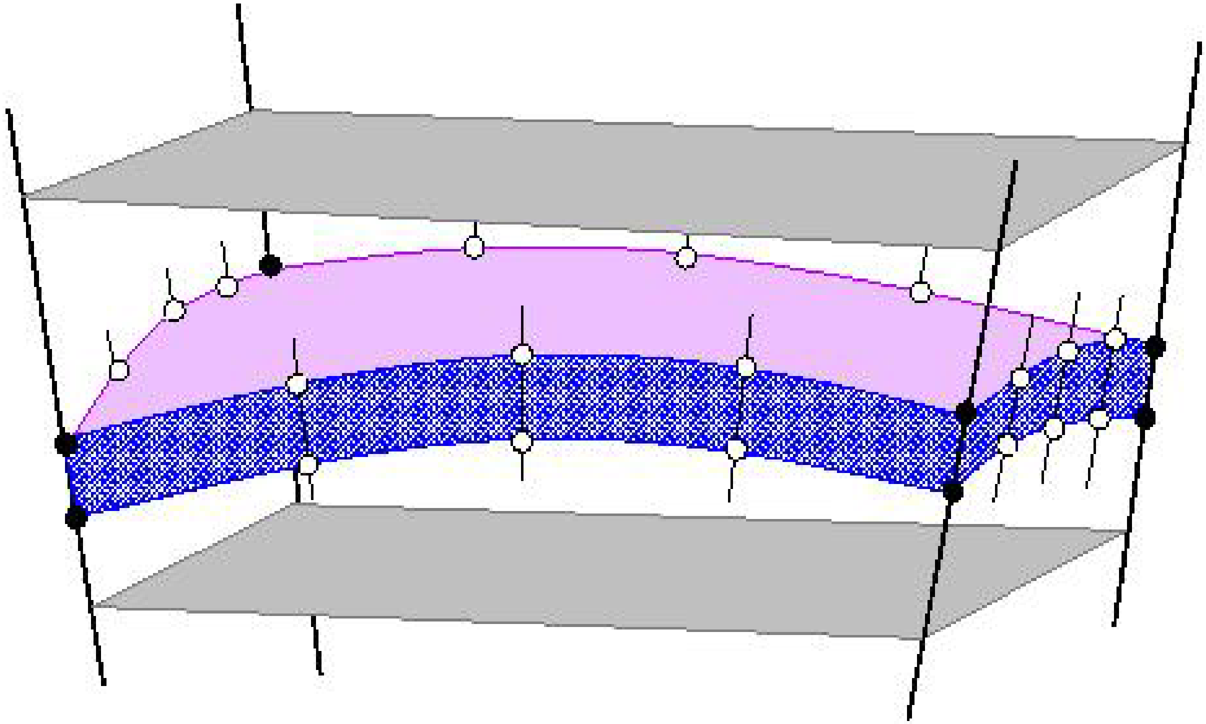
C^1 Shell Elements within a Cube

C^1 Quad Shell Surfaces can be built in a similar way, by defining functions over a cube



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