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B6, B4.18

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Mathematics and Information, Exercise sheet 10

Aufgabe 1: *(8 Punkte)*

a) Let A be a matrix with n rows and $m \leq n$ columns, and let $A = U\Sigma V^T$ be its singular value decomposition. Show that the first m columns of V are determined uniquely by the corresponding colums of U!

b) What is the image of the sphere

$$\mathbb{S} = \{x \in \mathbb{R}^m \mid \|x\| = 1\}$$

under the linear map φ: $\begin{cases} \mathbb{R}^m \to \mathbb{R}^n \\ x \mapsto Ax \end{cases}$?

Aufgabe 2: *(8 Punkte)*

a) Compute the singular value decomposition of the matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 3 & 0 & 4 \end{pmatrix}$!

b) Which 2×3-Matrix of rank 1 is closest to A in the sense of the Frobenius norm?

Aufgabe 3: *(4 Punkte)*

Let $A \in \mathbb{R}^{m\times n}$ be a matrix of rank r with singular values $\sigma_1 \geq \cdots \geq \sigma_r$. The L^2-norm $\|B\|_2$ of a matrix $B \in \mathbb{R}^{m\times n}$ is the maximal length of the vectors Bx for vectors x of length one. Show that the rank of $A + B$ ist at least r if $\|B\|_2 < \sigma_r$!

Can be submittel till Thursday, November 21, 2018 at 11.55h