

October 18, 2018

Mathematics and Information, Exercise sheet 7

Problem 1: (6 points)

In a race with three horses, a bookmaker offers to return three times the wager for all bets on the winning horse. The probability that the first horse wins is 50%; for the two other ones it is 25%. Determine the log-optimal portfolio and the expected doubling rate!

Problem 2: (8 points)

Suppose there are bookmakers offering different odds. Then, by choosing for each horse i the one with the best odds o_i , you might find that $S = \sum 1/o_i < 1$. In this case a *Dutch book* strategy is possible. Its portfolio sets $b_i = 1/So_i$.

- What is the gain or loss with this portfolio, if the i^{th} horse wins?
- Suppose, there are only two horses. One bookmaker offers $o_1 = 1.5$ on the first horse, another offers $o_2 = 6$ on the second. Determine the *Dutch book* portfolio!
- Assume that both horses are equally likely to win. Which doubling rate does one get with the *Dutch book*, and what's the optimal doubling rate?
- What's the probability of a loss with the log-optimal portfolio, and what is it for the *Dutch book*?

Problem 3: (6 points)

For the game *Lotto light*, invented for this exercise, every player chooses a number from one to eight. Every week such a number is drawn by a fair process, where each number has equal likelihood to be drawn. All the bets are accumulated and equally distributed among the winners. Empirical investigations showed that betters have different probabilities to choose one of the numbers; the probability to choose i is p_i given in the following table:

i	1	2	3	4	5	6	7	8
p_i	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{1}{16}$

- Which strategy should a player choose if he intends to place a lot of bets?
- Which expected doubling rate does he achieve with this strategy?
- If he starts with an initial stake of 100 Euros: How many rounds does it take till he is a millionaire, assuming he can bet any amount?