

September 14, 2018

Mathematics and Information, Exercise sheet 2

Problem 1: (4 points)

- a) Show that the intersection of two convex sets $A, B \subseteq \mathbb{R}^n$ is convex!
- b) What about their union $A \cup B$ and their symmetric difference $A \Delta B$?

Problem 2: (3 points)

Some message uses an alphabet A consisting of n letters; their probabilities are

$$p_1 < p_2 < \dots < p_n.$$

Show that the entropy strictly increases, if the probabilities of $m \geq 2$ letters are replaced by the average probability of these m letters!

Problem 3: (6 points)

- a) An alphabet A contains n letters; the probability of each letter is at least $1/2n$. Compute the minimum and the maximum for the entropy of a source using such an alphabet!
- b) What happens, if instead we know that each letter occurs with a probability $p \leq 2/n$?

Problem 4: (7 points)

- a) Suppose there are eleven balls of which at least ten have the same weight. How can you determine by three weighings with a balance scale if there is a ball with a different weight and if so, if it is heavier or lighter than the rest?
- b) What is the maximal number of balls for which two weighings suffice?
- c) What is the maximal number if it is known that exactly one ball has a different weight?
- d) What is the maximal number if it is known that one ball is heavier than the rest?