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B6, B4.18

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Mathematics and Information, Exercise sheet 1

Problem 1: (*8 points*)

a) A message source produces signs form an alphabet A consisting of six letters with respective probabilities $1/2, 1/4, 1/8, 1/16$, and twice $1/32$. Compute its SHANNON entropy!

b) Construct a binary encoding of these letters such that any sequence of letters from A has a unique encoding, and the expected value of the code length is as small as possible!

c) What changes in *a)* and *b)*, if all elements of A have equal probability?

Problem 2: (*6 points*)

A letter produced by a message source is more surprising if it has lower probability. If we want to describe this by a function $S(p)$, we should therefore demand that $S(p)$ is continuous and strictly decreasing. We also demand that surprises add up, i.e.

$$S(pq) = S(p) + S(q)\,.$$

a) Show that for every such function $S(p)$ there exists an $a > 1$ such that $S(p) = -\log_a p$!

b) Given a source with alphabet A and probabilities $p_1, \ldots, p_n$, what is the expected value of $S(p)$?

Problem 3: (*6 points*)

A message source with alphabet $A = \{0, 1, 2, \ldots, 10\}$ produces letters according to the following rule: It throws a fair coin at most ten times, but stops, if the coin shows *head.* If this happens in the i^{th} experiment, $i \le 10$, the letter i is sent; if it never happens, 0 is transmitted. Compute its entropy!

Hint: $\displaystyle\sum_{i=0}^{n} iq^i = \frac{nq^{n+2} - (n+1)q^{n+1} + q}{(1-q)^2}$

Can be submittel till Thursday, September 13, 2018 at 11.55h