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B6, B4.18

September 7, 2018

Algebraic Statistics, Exercise sheet 1

Problem 1: (*12 points*)

a) Write $S = V(X^2 - Y^2, X^2 - 1) \subset \mathbb{R}^2$ as a finite set of points!

b) Show that it is not possible to determine all the coefficents if the most general quadratic model

$$P = aX^2 + bXY + cY^2 + dX + eY + f$$

if only the values of P in the points of S are known!

c) Identify those types of quadratic models that can be identified by their values on S !

d) Now let $T = \{(0,0), (0,1), (0,-1), (1,1)\,(-1,-1)\} \subset \mathbb{R}^2$ be a given sample. Find (by trial and error) two polynomials $f, g \in \mathbb{R}[X,Y]$ such that $T = V(f,g)$!

e) Can the coefficents of P from *b)* be computed from the values of P on T ?

f) If not, find those simpler quadratic models for which this is possible!

Problem 2: (*2 points*)

a) Let R be a commutative ring. Show that the intersection of arbitrarily many ideals of R ist again an ideal!

b) Does the same hold for their union?

Problem 3: (*6 points*)

An ideal I in a domain R is called a *principal ideal,* if there exists an $f \in R$ such that $I = \{fg \mid g \in R\}$ consists of all multiples of f. If every ideal in a ring R is a principal ideal, R is called a principal ideal domain.

a) Let $R = \mathbb{Z}$ and $a, b \in \mathbb{Z}$. Find numbers $c, d \in \mathbb{Z}$ such that $(a) \cap (b) = (c)$ and $(a, b) = (d)$!

b) Show that $\mathbb{Z}$ is a principal ideal domain!

c) Now let $R = \mathbb{R}[X_1, \dots, X_n]$ and $f_1, \dots, f_m \in R$. Show that $V(f_1, \dots, f_m) = V(f_1^2 + \cdots + f_m^2)$!

d) Is $\mathbb{R}[X_1, \dots, X_n]$ a principal ideal domain?

Can be submittel till Wednesday, September 12, 2018 at 11.55h